EXPERIMENTAL SCIENCE

I. PHYSICS

CAMBRIDGE UNIVERSITY PRESS

C. F. CLAY, Manager

LONDON : FETTER LANE, E.C. 4

LONDON: H. K. LEWIS AND CO., Ltd.,
136 Gower Street, W.C. 1

LONDON: WHELDON & WESLEY, Ltd.,
2–4 Arthur St., New Oxford St., W.C. 2

NEW YORK: THE MACMILLAN CO.

BOMBAY }
CALCUTTA } MACMILLAN AND CO., Ltd.
MADRAS }

TORONTO: THE MACMILLAN CO. OF CANADA, Ltd.

TOKYO: MARUZEN-KABUSHIKI-KAISHA

EXPERIMENTAL SCIENCE

I. PHYSICS

BY

S. E. BROWN, M.A. (Cantab.), B.A., B.Sc. (Lond.)

HEADMASTER OF LIVERPOOL COLLEGIATE SCHOOL, FORMERLY
SENIOR SCIENCE MASTER AT UPPINGHAM

Cambridge:
at the University Press
1923

First Edition 1913
Reprinted 1914
Revised Edition 1915
Reprinted 1916, 1917, 1918 (*twice*), 1919, 1920 (*twice*), 1922,
1923

PRINTED IN GREAT BRITAIN

PREFACE

THE following introductory course of Experimental Science is divided into two parts—I Physics, II Chemistry—the former containing sections on (1) Measurement, (2) Hydrostatics, (3) Mechanics and (4) Heat.

The book is based on the practical experience of practical teachers and every care has been exercised to provide a set of lessons which will not only educate the pupil but also maintain interest and attention.

In the particular Secondary School where the course has been put to the test by about 450 boys working in 16 classes (Forms III and IV), it has been found that the average pupil can, during two years, complete nearly the whole of Part I, Physics, by working for four periods per week, two of the periods running consecutively for laboratory practice. When, however, only four periods per week are allotted to science work, a part of the Section on Mechanics is deferred until the third year.

Five of the writer's colleagues, specialists in Mathematics and Science, have collaborated with him in reading proof-sheets and in giving the benefit of their experience and kindly criticism ; and he wishes to gratefully acknowledge the help of Mr L. Caldecott, B.A., B.Sc., Mr T. V. T. Baxter, M.A., B.Sc., Mr W. R. Cooper, B.A., Mr A. C. Baggley, M.A. and Mr J. M. Moir, M.Sc.

The writer is especially indebted to Mr Lawrence Caldecott, Senior Science Master at the Liverpool Collegiate School, who has prepared, and has tested in his classes, the carefully graduated examples which form, in the writer's

opinion, one of the most important features of the book. Continued and systematic working through these examples is essential and will ensure success for the pupils. Mr Caldecott has also given untiring assistance in contriving details of laboratory method which will help beginners to obtain with certainty accurate results even in difficult experiments such as finding the Coefficient of Expansion of a Gas and the Latent Heat of Vaporization of Steam.

The writer acknowledges with thanks the permission of Messrs F. E. Becker & Co., Hatton Wall, Hatton Garden, E.C., to use diagrams Nos. 90, 91, 107, 138, 142–6, 160, 169, 178*a*, 181, 188, and 197.

The author believes that the experiments chosen throughout the book are presented in a newer and simpler form perhaps than heretofore, and that this alone should justify the publication of another text-book on Experimental Science.

The following are among the examinations which may be successfully taken after carefully working through Experimental Science, Parts I and II: Oxford and Cambridge Local Examinations—Experimental Science (Junior), Chemistry (Junior), Heat (Junior, and Senior); Army Qualifying Examinations, Naval (Boy Artificers), Board of Education, College of Preceptors, and Examinations for Junior County Scholarships, which require Physics, Chemistry and Practical Mathematics.

S. E. BROWN.

LIVERPOOL,
Jan. 1917.

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SECTION I.

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CHAPTER I.

MEASUREMENT OF LENGTH, ANGULAR MEASUREMENT, TIME.

1. Measurement of Length.

Unit of measurement. A halfpenny is 1 *inch* in diameter. If five halfpennies placed in line, edge to edge, stretch exactly across a book, we should then say that the width of the book is 5 *inches*. An *inch* would be here the *unit of measurement.*

In ancient days a barley corn, a span and a cubit (the length of the fore-arm) were three among many units of length. Confusion arose because the unit varied.

A golfer was once asked, "How far is it to the post-office?" He replied, "A full drive, a brassy shot and an approach with the light iron"; a boy might have said that it was "eight times as far as he could throw the cricket ball." If golfers always drove a ball equal distances or if all boys had exactly the same range in throwing, we *might* adopt one of these lengths as a **unit**: but it is not probable, for *units must be definite*, simple and well-known.

The British Unit of Length is the **standard yard,** which is (by Act of Parliament) the distance (at 62° F.) between the lines on two gold plugs let into a certain bronze bar. This *standard yard* is deposited at the Board of Trade Offices and there are copies kept at the Houses of Parliament, the Royal Society, the Royal Mint and the Greenwich Observatory.

1 yard = 3 feet = 36 inches,

1760 yards = 1 mile.

The Metric Unit of Length or **standard metre** was fixed by the government of France (1799) to be the length of a certain platinum rod at 0° C. Accurate copies of the *standard metre* are kept.

It was intended that the length of this rod should be one forty-millionth part of the circumference of the earth measured on the great circle (meridian) through Paris and the North and South Poles. Later observation has proved that the calculation was not quite accurate.

1 metre = 39·37... inches,

1 metre (m.) = 10 decimetres (dm.),

= 100 centimetres (cm.),

= 1000 millimetres (mm.).

Therefore, by dividing by 1000,

0·001 metre = 0·01 decimetre = 0·1 centimetre = 1 millimetre

Hence we can *alter the unit by altering the decimal point.*

Thus 4·321 metres = 43·21 dm. = 432·1 cm. = 4321 mm.,

and we can read the quantities thus:

4 metres 3 decimetres 2 centimetres 1 millimetre.

1000 metres = 1 kilometre = $\frac{5}{8}$ mile (approximately),

1 inch = 2·54 centimetres.

13 inches = 33 centimetres.

***Ex.** i. You are provided with a ruler divided into *centimetres* on one edge and *inches* on the other. Copy the first 3 inches and 8 centimetres of this ruler accurately in your note book

***Ex.** ii. We notice that the *Metric* is a *decimal* system [Latin, *decem* = ten].

10 millimetres = 1 centimetre,

10 centimetres = 1 decimetre,

10 decimetres = ...

* An asterisk * *prefixed* indicates that the Exercise or Experiment is suitable for the whole class to perform.

N.B. **Ex.** = **Exercise.** **Exp.** = **Experiment.**

Continue this table in your note book, remembering the order of the units: *millimetre* (mm.) [Lat. *mille* = 1000], *centimetre* (cm.) [Lat. *centum* = 100], **decimetre** (dm.) [Lat. *decimus* = a 10th]; metre (m.); *dekametre* (Dm.) [Greek, *deka* = 10], *hectometre* (Hm.) [Gr. *hekaton* = 100], *kilometre* (Km.) [Gr. *chilioi* = 1000]. The sub-multiples (fractions) of a metre have names derived from Latin, the prefixes of the multiples are derived from Greek.

How to use a scale for measuring lengths.

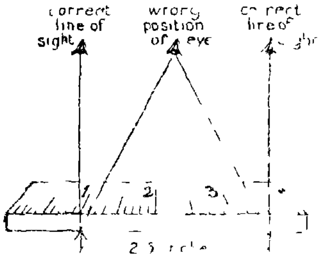


Fig. 1 *a*.

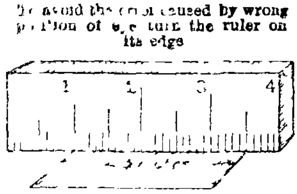


Fig. 1 *b*.

(1) Put the ruler on its edge so that the *scale is as near as possible* to the object to be measured (Fig. 1 *b*)

(2) See that the line of sight from the eye to the scale is always at *right angles* to the scale (Fig. 1 *a*) [See also *Exp. Chemistry*, § 8, *Parallax Error*.] To avoid Parallax Error place the scale so that the graduation marks touch the line to be measured (Fig. 1 *b*).

(3) Put one end of the object to be measured exactly opposite to the first *unit mark* (1 cm. or 1 inch) and remember in reading the scale at the other end of the object to *subtract* 1 from the number of units. See Fig. 2

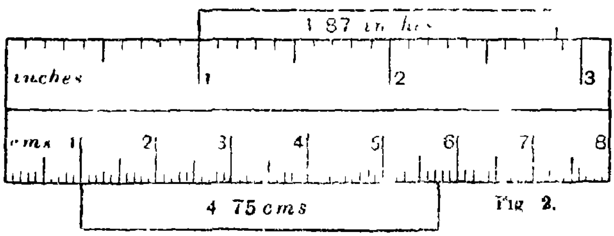


Fig 2.

(4) ***Estimate by the eye*** to the 2nd decimal place. Thus in Fig 2 the length of the object = 1·87 inch,
" " " = 4·75 cm.

*Practical Exercises on Measurement of Length

1. Draw lines 1, 2, 3 and 4 inches long. Measure each line in millimetres and calculate the number of millimetres in one inch. Take the mean of your four answers.

2. Draw a line 2·4 inches long. Measure it in centimetres and calculate the number of centimetres in one inch.

3. Draw a line 8·9 centimetres long. Measure it in inches.

4. Draw a line *AB* 4 centimetres long and from *B* draw at right angles to *AB* a line *BC* 3 centimetres long. Measure *AC* in centimetres.

5. Measure the width of the bench in decimetres and in feet and find the number of decimetres in one foot.

6. Measure the length of the bench in yards and in metres and calculate the number of inches in one metre.

2. Measurement of Curved Lines.

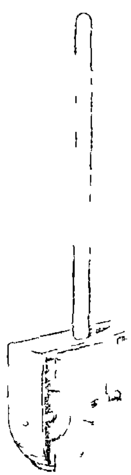


Fig. 3.

*Ex. i. **By thread.** Draw a circle of 5 cm. radius. Mark any point *P* on the circumference, then place the end of a piece of cotton at this point and stretch the thread on the circumference, a short piece at a time round the circle, until the point *P* is reached again. Measure the length of thread required on your scale. Repeat three times and take the mean of your results.

Note. The **mean** or **average** is obtained by dividing the sum of your results by the number of observations taken.

*Ex. ii. **By dividers.** Measure the same circumference using a pair of finely pointed "dividers," keeping the points of the compasses exactly 0·5 cm. apart.

Ex. iii. **By the Opisometer** (lit. "backward measurer"). An *opisometer* is a small wheel with a milled or finely toothed edge set on an accurate screw as axle (Fig. 3). The wheel carries a mark which is adjusted to a pointer on the frame. The wheel is then made to travel along the curve to be measured and the number of revolutions are counted. It is then lifted from the curve and run backwards for the same number of revolutions along a *scale* until the pointer is readjusted to its original position.

Length of circumference of ⊙ = 31·4 cm.

3. To find the Ratio:

$$\frac{\text{Circumference of a Circle}}{\text{Diameter of the same circle}} = \pi.$$

***Ex.** i. Tabulate all the results of § 2, including a column for the ratio $=\pi$.

***Ex.** ii. Make a small mark on the edge of the given cylinder. **Roll the cylinder** along a piece of paper and measure the distance in cm. between the two points where the mark consecutively touches the paper. Place the cylinder on the scale and measure its diameter in cm.

Work out the ratio (π),

$$\frac{\text{circumference (cm.)}}{\text{diameter (cm.)}} = \pi.$$

***Ex.** iii. **Wrap a** thin piece of **paper** once tightly **round the cylinder.** Prick a hole through the paper and measure the distance in cm. between the pin-pricks on the unrolled paper to obtain the circumference. Find the ratio (π)

***Ex.** iv Wrap the piece of thread 10 times tightly round the cylinder without overlapping. Measure the length in cm. of thread required and divide by 10 to obtain the circumference. Find the ratio (π).

From these exercises we find that—

In a circle the ratio $\dfrac{\text{circumference}}{\text{diameter}}$ = a constant,

$$\begin{aligned} &= \pi \text{ (pronounced pie)}, \\ &= 3\ 14159\ldots, \\ &= 3\tfrac{1}{7} = \tfrac{22}{7} \text{ (nearly)}. \end{aligned}$$

If r = radius of a circle, $2r$ = its diameter, then,

$$\frac{\text{circumference}}{2r} = \pi.$$

$\therefore$ the **circumference of a circle** $= 2\pi r$.

4. Other Measuring Instruments.

***Ex.** i. **Outside and Inside Callipers** (Fig. 4). Use the end AB for measuring the diameter of a cylinder and a sphere. Verify by finding the circumference and dividing by π.

The end CD may be opened inside a test tube and the internal diameter of the tube found by measuring CD on a suitable scale.

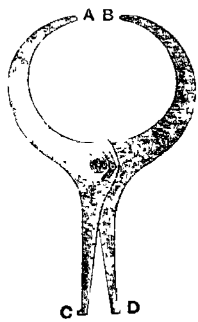


Fig. 4.

*Ex. ii. **The Wedge.** Cut a piece of stiff paper, as straight as possible, with a sharp knife. Divide its edge *AB*, with your scale, into 10 centimetres (Fig. 5). Erect perpendiculars at the centimetre divisions. Make *BC* exactly = 1 cm. and cut off the wedge *ABC* with your knife. Measure the perpendiculars and note that their lengths in millimetres equal the corresponding number on the cm. scale. Use the wedge for measuring the internal diameter of glass tubing.

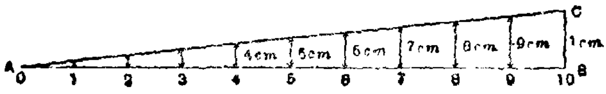


Fig. 5.

*Ex. iii. Fig. 6 is a copy of a diagonal scale. Notice that a series of "wedges" would be made by erecting perpendiculars at the division marks of *CD*. Measure *CD*. Count the number of divisions in *DC* and *CA*. Use this diagonal scale and your dividers and mark off in your note book lengths = 0·2; 0·6; 1·3; 1·7; 1·04, 1·09; 1·26; 1·68; 1·97 inches.

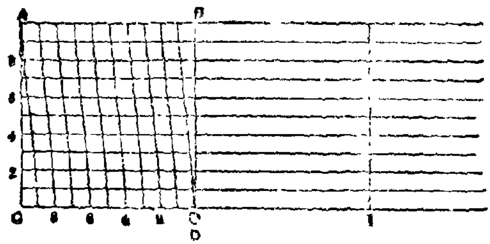


Fig. 6.

5. The Screw.

From a piece of "inch" squared paper cut a paper wedge as in Fig. 5, but make $AB = 12$ inches, $BC = 3$ inches. Mark the perpendiculars at the inch divisions along *AB*. Then roll the paper round a pencil, laying *BC* along the pencil; the edge *AB*

must not overlap when wrapped (Fig. 7). Gum the point A down and observe that you have made a *screw* with the edge AC. Count the number of turns (here $8\frac{1}{6}$) and divide into 3 inches, the length of BC. This gives the *pitch* or distance between consecutive threads, which will be the same throughout if the shaft of pencil tapers[1] to compensate for the increased thickness of the rolled paper. Apply the principle of the wedge and notice that it is possible to measure distances along the pencil with accuracy by noting the number of complete turns and the fraction of a turn which (say) length XY passes over when laid along the screw.

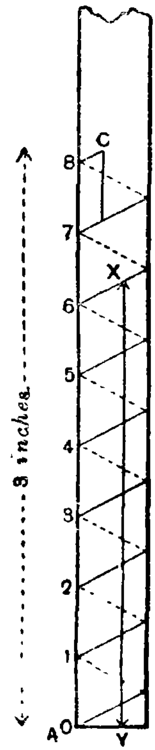


Fig. 7.

Length of XY = pitch × number of turns

$$= \frac{3}{8\frac{1}{6}} \times 6\frac{1}{3} = \frac{18}{49} \times \frac{19}{3} = 2{\cdot}33 \text{ inches}.$$

6. The Screw Gauge.

A screw is the most accurate instrument for measuring small lengths. The distance between the threads (the *pitch* of the screw) is found by dividing the distance travelled by the end of the screw when turned several times by the number of turns. Thus in the particular instrument shown in Fig. 8, if the screw $EFHG$ is turned 20 times, the face of E moves through 1 cm.; therefore the pitch $= \frac{1}{20}$ cm. $= \frac{1}{2}$ mm. The circular head of the screw HG is divided, on the circle H, into 50 divisions. If the screw is turned through say 17 divisions, E will travel $\frac{17}{50}$ of the pitch, *i.e.* through

$$\frac{17}{50} \times \frac{1}{2} = \frac{17}{100} = {\cdot}17 \text{ mm}.$$

In using a screw gauge, first screw up until the faces D and E touch: the arrow on H should then point to O on the frame. If this is not the case, note the *fraction of a turn* through which the milled head

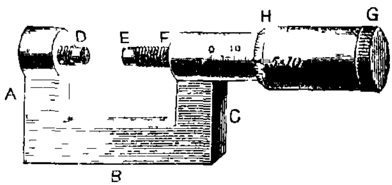


Fig. 8.

[1] Use sandpaper to obtain the required "taper."

must be *unscrewed*[1] to bring the arrow on H pointing to O. This fraction is called the **zero error** and must be *added* to the result.

Ex. i. To find the diameter of the given wire.

Enter your results as follows:

Zero error	$\frac{11}{50}$ turns	$=\frac{11}{50}\times\frac{1}{2}$ mm.	$=0{\cdot}11$ mm.
Number of complete turns	3 ,,	$= 3\times\frac{1}{2}$ mm.	$=1{\cdot}50$ mm.
Additional fraction of turn	$\frac{47}{50}$,,	$=\frac{47}{50}\times\frac{1}{2}$ mm.	$=0{\cdot}47$ mm.
		$\therefore$ diameter of wire	$= 2{\cdot}08$ mm.

Ex. ii. Find the average thickness of the piece of *plate glass* and the diameter of the *steel rod* provided.

7. The Spherometer.

The spherometer is constructed on the same principle as a screw gauge. It is used for measuring the radius of curvature of the faces of lenses which are usually segments of spheres. The zero position and *zero error* are found by placing the spherometer on a sheet of glass so that it stands on its rigid three legs (Fig. 9). The screw is then turned until its point just touches the glass. If the zero (0) on the divided circle is not then opposite the upright scale, the screw is turned until this latter position is obtained and the fraction (the *zero error*) is noted down. There is now a space between the glass and the point of the screw, represented by the zero error, and the length of this space must be added to results, as directed in the use of the screw gauge.

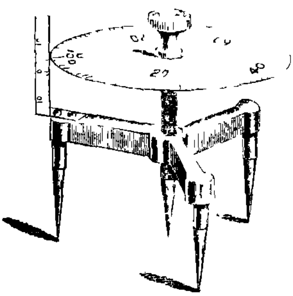


Fig. 9.

8. The Vernier.

Fractions of a scale division may be *estimated by the eye* with considerable accuracy after some practice. By means of a *vernier*, however, we can obtain the measurement with precision. Fig. 10 represents a scale in cm. below and a moveable scale or *vernier* above, by which readings to one-tenth of a scale division may be taken. If we are measuring the length of a bar, we lay it on the scale, one of its ends being at the zero and the other between (say) 46 and 47 on

[1] This method always brings the zero error *positive*.

the scale. The vernier is then pushed along the scale until its zero, indicated by the left-hand arrow, is exactly opposite the end of the bar. We now look along the vernier and find *which of its*

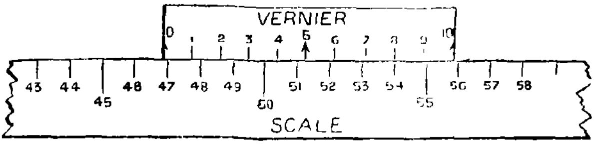


Fig. 10.

divisions is opposite to a scale division. No. 8 is opposite 54. The length of the rod is then recorded as equal to 46·8 cm.

By counting, we find that *this vernier contains* 10 *divisions* (between the arrows) and that these 10 vernier divisions = 9 scale divisions.

Therefore

each vernier division = $\frac{9}{10}$ scale division = $\frac{9}{10}$ cm.,

i.e. „ „ „ is $\frac{1}{10}$ cm. short of a scale division;

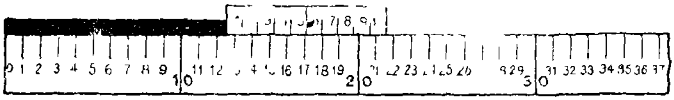
therefore if the 1st vernier division is opposite to a scale division, there is a space of $\frac{1}{10}$ or ·1 cm. between the zero of the vernier and the nearest division on the scale. If the 2nd vernier division is opposite to a scale division this space equals $\frac{2}{10}$ or ·2 cm. But the 8th vernier division is opposite to a scale division, therefore this space equals $\frac{8}{10}$ or ·8 cm.

If the scale is in inches and we wish to construct a vernier reading to $\frac{1}{8}$ inch, the vernier must be made 7 inches long and be divided in 8 equal parts.

If the inch scale is divided into eighths and we wish to construct a vernier reading to $\frac{1}{64}$ inch, the vernier must be $\frac{7}{8}$ inch long and be divided into 8 equal parts.

***Ex.** With the inch scale (divided into tenths) use the piece of card-board provided to construct a vernier reading to $\frac{1}{100}$ or ·01 inch.

Diagram to illustrate the use of scale and vernier.



Length of rod = 12·6 scale divisions.

FIG. 11.

Memoranda on the use of instruments fitted with verniers.

(1) Count the number of divisions on the vernier, and

(2) The number of scale divisions to which these are equal.

Find the relative position of vernier to scale by sliding the vernier to the zero position; *e.g.* in Fig. 12, it is evident that, to find the distance between the jaws of the calipers, the scale on the calipers must be read from the zero on the vernier.

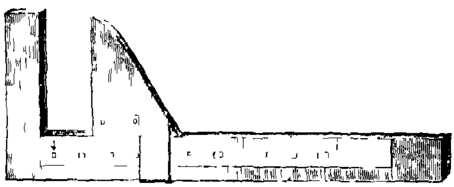


FIG. 12.

9. Graphic Representation of Length.

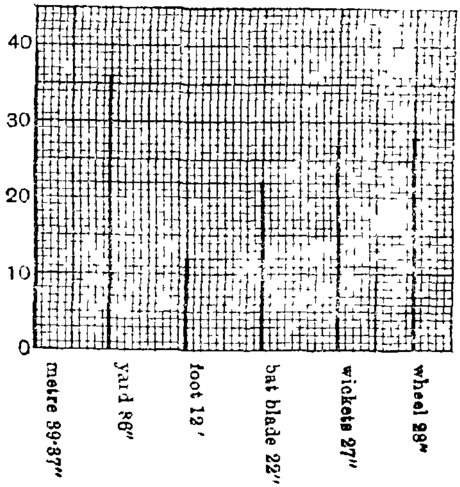
By means of *straight lines drawn to scale* it is easy to compare various lengths, especially if they all are expressed in the *same unit*.

Thus in Fig. 13 a metre, a yard, a foot, the length of a cricket bat blade, the height of the wickets and the diameter of a bicycle wheel are shown by *straight lines* on squared paper drawn to *scale* 1 *mm. to the inch*.

Length has one dimension and may be **represented by a straight line.**

*Ex. Express the following lengths in *yards* and represent them

graphically to scale on squared paper: (1) a chain (22 yards) or the length of the cricket pitch, (2) 100 yards, (3) hectometre, (4) 5 chains, (5) a furlong



Scale: 1 mm. = 1 inch.

Fig. 13.

(10 chains), (6) a quarter mile, (7) the length of s.s. Olympic (880 feet). (8) height of St Paul's (363 feet).

10. Angular Measurement.

When two straight lines meet at a point they are said to *contain an angle*. When a line sweeps round in a plane about one of its extremities it is said to *describe an angle*. This *turning of a line* about one of its ends is seen in a watch. Imagine the minute hand to point continuously to 12 o'clock. The hour hand is allowed to sweep round in one direction, firstly to 3 o'clock, secondly to 6 o'clock, thirdly to 9 o'clock, and fourthly to 12 o'clock again. The hour hand has accordingly described 1, 2, 3 and 4 right angles (Fig. 13 *a*).

Each right angle is divided into	90	*degrees*	=	90°.
∴ an angle of 2 right angles contains	180	„	=	180°,
„ „ 3 „ „ „	270	„	=	270°,
„ „ 4 „ „ „	360	„	=	360°.

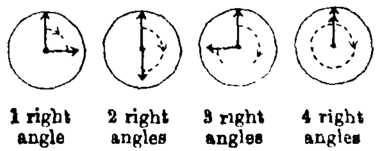


Fig. 13 *a*.

If the hour hand moves from 12 o'clock to 1 o'clock it describes an angle of $\frac{1}{3}$ of a right angle = 30°, if from 1 o'clock to 3 o'clock, it describes 60° and so on.

Ex. *How many (a) degrees, (b) right angles, does the hour hand describe* in moving (1) from 1 o'clock to 4 o'clock; (2) from 2 o'clock to 4 o'clock; (3) from 4 o'clock to 8 o'clock; (4) from 10 "past" to 24 "past"; (5) from 17 "past" to 55 "past" (5 "to").

The sum of all the angles about one point in a plane = 4 right angles or 360°.

A right angle contains 90°.

1 degree (1°) = 60 minutes (60′).

1 *minute (1′) = 60 seconds (60″).*

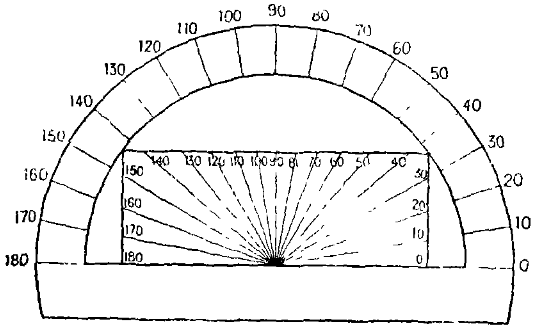


Fig. 13 *b*.

Protractor. Fig. 13 *b* shows two kinds of protractors superposed. Draw the protractor provided and explain its use by a diagram.

Ex. i. Double a piece of paper along one of its edges. Halve the angle so formed by folding and again halve the angles so formed. Measure the number of degrees in each different angle in order of magnitude.

Ex. ii. In a yacht race over a triangular course *ABC*, a yacht sailed from *A* to *B* (3 miles), from *B* to *C* (7 miles), from *C* to *A* (9 miles) and again from *A* to *B*. Draw the course to scale and measure with your protractor the angles through which the yacht turns at *B*, *C* and *A*.

11. Measurement of Time.

The Sidereal Day. The Earth rotates on its axis once a day. Imagine that we have erected in a *vertical* position a smooth *plane* sheet of metal the lower edge of which points *north* and *south*. By looking along the surface of this sheet we should be able to see on clear nights that the pole star is always in the plane of which the surface of the sheet is but a part and that the other stars seem to cross it as they move round the pole star. We know however that the stars are fixed and that the Earth is revolving on an axis which points to the pole star. We can imagine this vertical *plane* continued until it passes through the N. and S. Poles—it is called the plane of the *meridian*. If, night after night, we timed consecutive crossings or *transits* of any particular star through this plane we should find that there was always an interval of 23 hours 56 min. 4 secs. This interval of time is called a **sidereal day.**

The Mean Solar Day. The Earth takes a year to travel round the Sun. It has turned on its axis in space 366 times, but to an observer on the Sun 365 times. If we time the intervals of the Sun's transit across the meridian [Solar Days] we find that they vary; for, although the meridian plane is sweeping through equal angles in equal times, the Earth is not moving at a uniform speed in its path round the Sun. The average length of the Solar Days is called the **Mean Solar Day** which is divided into 24 hours, or 24 × 60 minutes, or 24 × 60 × 60 seconds.

The Unit of Time is the *mean solar second*

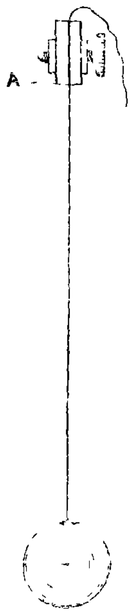


Fig. 14

12*. The Pendulum.

Exp. You are provided with a leaden ball about an inch in diameter to which a strong piece of thread is attached by a small ring sunk into the lead. Clamp two pieces of wood together and between them secure the thread (Fig. 14). Fix the point of suspension firmly to the bench. Mark a chalk line on the floor vertically below the point of suspension. Let the pendulum swing so that its amplitude (=half the path traversed by the bob) is not greater than ⅙ of its length.

(1) Using the seconds hand of your watch, take the time of 50 *vibrations* or intervals between consecutive crossings over the chalk line. Remember to count 0, 1, 2, 3, .. where 0 is said at the beginning of the time observed. Repeat three times and take the *mean* or average and hence calculate the time of one vibration.

Note that the *time of vibration is independent* of the *amplitude* as long as the amplitude is small.

(2) Measure the *length* of the pendulum (AB) from the point of suspension to the centre of the bob.

(3) Lengthen the pendulum and repeat your observations.

Record your results as follows:

Length ($=L$)	Average Time of 50 Vibrations	Time of one Vibration ($=T$)	Square of Time of Vibration ($=T^2$)	$\frac{L}{T^2}$
40 cms.	31·5 secs.	0·63 sec.	0·397 sec.	101
60 ,,	38·5 ,,	0·77 ,,	0·593 ,,	101
80 ,,	45·0 ,,	0·90 ,,	0·810 ,,	99
100 ,,	49·5 ,,	0·99 ,,	0·980 ,,	102
120 ,,	55·0 ,,	1·10 ,,	1·210 ,,	99

13*. Graphic representation of two varying quantities, e.g.

$$\frac{\text{Length of Pendulum}}{\text{Time of Vibration}}.$$

N.B. * placed *after* the number of a paragraph means that the paragraph may be omitted for the *first time of reading*.

We have learnt in § 9 how to compare lengths graphically by means of lines drawn to scale on squared paper. Let us extend the method to show at a glance how the *time of vibration depends on the length of the pendulum*.

Draw a *horizontal* line OX (called the axis of X). Mark off from O lengths to represent the *time of vibration in seconds* (Fig 15).

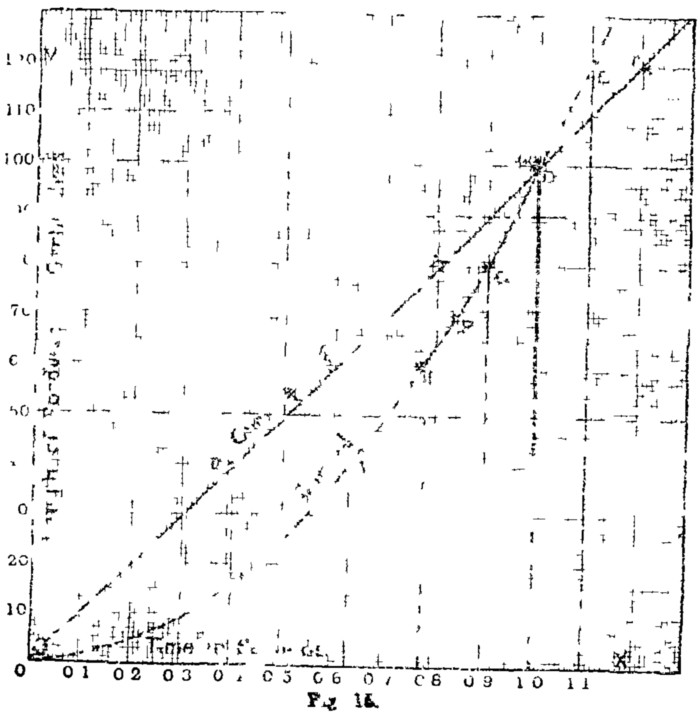


Fig. 15.

At O draw a *vertical* line OY (called the axis of Y) and divide it so that we can measure vertically (from the axis of X) the varying length of the pendulum.

The centre of the bob of the pendulum is supposed to be always on the axis of X and is placed at a distance from O representing the time of vibration. A vertical line is then drawn proportional to the length of the string. For instance at a distance of 6·3 cm. from O along OX corresponding to a time of vibration of 0·63 sec. a perpendicular is drawn of length 4·0 cm. corresponding to a pendulum 40 cm. long. The point of suspension is represented by A, the extremity of this perpendicular. Similarly the point B is obtained for a pendulum vibrating in 0·77 sec. and of length 60 cm. A series of points A, B, C, D and E are obtained which may be joined by a line or *curve.*

Intermediate values may be obtained from the curve, *e.g.* the point P indicates that a pendulum of 70 cm. length would vibrate in 0·84 sec.

The "*slope of the curve*" at any point is found by drawing a *tangent* (or line touching the curve) at that point. The "slope" shows us the increase in length for a given small increase in the time of vibration. We see that the "slope of the curve" becomes steeper as the time of vibration increases.

The process of finding similarly a series of points from two sets of observations is called "*plotting a curve.*" If we take *the squares of the times of vibration* (column 4) with the corresponding *lengths* of the pendulum to "plot a curve," we find that all the points (a, b, c, d, e) obtained lie nearly in a *straight line.* The "slope of the curve" is then constant, *i.e.*

$$\text{the fraction } \frac{\text{Length of pendulum}}{\text{Square of time of vibration}} \text{ is always the same.}$$

Or, the length of the pendulum is proportional to the square of the time of vibration.

The values obtained from our observations are shown in the last column, where

$$\frac{L}{T^2} = 100 \text{ (approximately).}$$

N.B. An *oscillation* is a complete "*to and fro*" movement: the time of *oscillation* = time of *vibration* × 2.

Practical Exercises on Angular Measurement.

1. Draw two lines crossing one another. Measure the four angles made by these lines and calculate their sum.

2. Draw a triangle, measure its three angles and find their sum.

3. Draw a rectangle with sides 15 cm. and 20 cm and measure the angles between its diagonals. (O. J.)

4 A parallelogram $ABCD$ has the sides AB, AC 11 cm. and 7 cm. long respectively, and the angle BAC is 120°. Draw the parm. finding D by a suitable construction with compasses. Find also (*a*) length of the perpendicular from D on AB, (*b*) length of shorter diagonal, (*c*) area of parm. (O. J.)

5. With centre O and a radius of 5 cm. describe a circle. Mark off on the circumference the following points in this order: A, C, D, E, B. Join AC, AD, AE, AO and BC, BD, BE, BO. By means of a protractor measure and record the angles thus formed at the points C, D, E and O, and state how much bigger the angle AOB is than the others. (O. J.)

6. $ABCD$ is a field in which AB is 60 yards, BC 80 yards and the angle at B is a right angle. The perpendicular from D on the diagonal AC is 40 yards. Draw an accurate figure (scale 1 inch = 20 yards) making the angle $ACD = 45°$ and calculate the area as the fraction of an acre. (O. J.)

7. Construct a right-angled triangle with hypotenuse $3\frac{1}{2}$ inches and one acute angle 20° greater than the other. (O. J.)

8. Draw a triangle of which two of the angles are respectively 90° and 30° and one of the sides 8 cm. long.

Determine by measurement what may be the lengths of the other sides. (O. J.)

Examples I a (Straight Lines).

1. How many cm. are there in (*a*) 1 dm., (*b*) 1 Dm., (*c*) 1 Hm., (*d*) 1 mm.?

2. How many metres are there in (*a*) 1000 cm., (*b*) 200 dm., (*c*) 10 Dm., (*d*) 10 cm.?

3. Express in cm. (*a*) 0·0067 km., (*b*) 507·65 m., (*c*) 0·65 mm., (*d*) 0·6 Hm.

4. Express in dm. (*a*) 7·105 km., (*b*) 2·7 cm., (*c*) 0·067 Dm., (*d*) 0·0561 cm.

5. Subtract (*a*) 0·0011 km. and 1055 mm. Give answer in cm.
(*b*) 7·612 km. and 6110 dm. Give answer in m.

6. Add 205 Dm., 26 dm., 16 Hm., 47 m., 500 mm.

Give answer in (*a*) metres, (*b*) kilometres.

7. How many pieces of wire 14 cm. long can be cut from a length of 0·7 Hm.?

8. A six-inch line measures 15·24 cm. Find the number of cm. in 1 inch and the number of inches in 1 cm.

9. If the height of the mercury in a barometer is 760 mm., what will it measure in inches?

10. Four values obtained for the length of a line were 260 cm., 2600·5 mm., 26 dm., 2599·5 mm. Find average length of line in mm.

---

11. Add together 755 cm., 50 Dm., 713 dm., 3 km.

Give answer in metres.

12. If a man walks at the rate of 3 miles an hour, how long will he take to travel 16·8 km.? (5 miles = 8 km.)

13. How many metre strides will be required to pace out a cricket pitch 22 yds. long?

14. A bar measures 24·13 cm. and 9·5 inches. Calculate the number of cm. in one inch.

15. A book containing 240 pages is 2·88 cm. thick. What is the average thickness of the pages in mm.?

EXAMPLES I B (CURVED LINES) $\pi = \frac{22}{7}$.

1. A spot of ink is placed on a ring, the radius of which is 1·26 cm. When the ring is rolled across paper the distance between two ink marks is found to be 7·92 cm. What value does this give for π?

2. Calculate the circumference of a circle the diameter of which is (*a*) 7″, (*b*) 8·5 cm., (*c*) 5·04 dm.

3. Find the diameter of a circle, the circumference being (*a*) 44 yds., (*b*) 15·4 cm.

4. Calculate the radius of a circle, the circumference of which is (*a*) 187 yds., (*b*) 89$\frac{3}{4}$ mm.

5. How many rings of diameter 3 cm. can be made from a piece of wire 5·5 m. long?

6. In travelling half a mile a bicycle wheel revolves 360 times. What is the diameter of the wheel?

7. How many flags will be required to mark out a circular racing track of 70 yds. radius if the flags are placed 10 yds. apart?

8. What distance in space would a spot at the equator pass through, owing to rotation, in 6 hours if the radius of the Earth at the equator be 3963·3 miles?

9. If the radius of a circular racing track measured from the centre to the inside edge be 70 yds., and the track be $3\frac{1}{2}$ yds. wide, how much farther will the man on the outside have to run than the man on the inside edge?

10. Given that electricity travels 186,400 miles per second, and that the Earth's equatorial diameter is 7926·6 miles, how many times could electricity travel round the equator in one second?

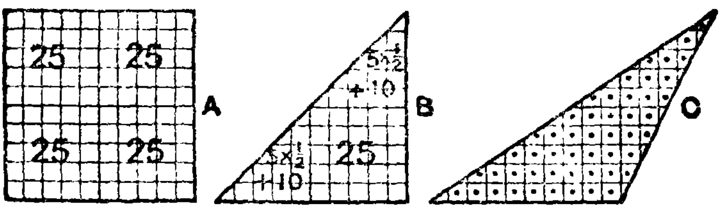
CHAPTER II.

MEASUREMENT OF AREA.

14. Area has two dimensions.

In measuring the length of a straight line joining two points it is only possible to take *one* observation. *Length* is said to possess *one dimension*. But an area has *breadth* as well as length. The area of a rectangular playground, for instance, 50 yards broad and 100 yards long, is found by considering that it is made up of 50 strips each one yard across and 100 yards long, *i.e.* the area or extent of the surface of the whole playground is 50×100 square yards. The rectangle is said to have *two dimensions*. We can imagine that a rectangle may be made with the same area as that of any plane figure that we can draw. Therefore any area has the same dimensions as a rectangle, *i.e.* an *area has two dimensions*.

15. Use of squared paper for measuring areas.



100 small squares

$12\frac{1}{2} + 25 + 12\frac{1}{2}$ $= 50$ squares.

$= 50$ squares, counting only the squares greater than half squares.

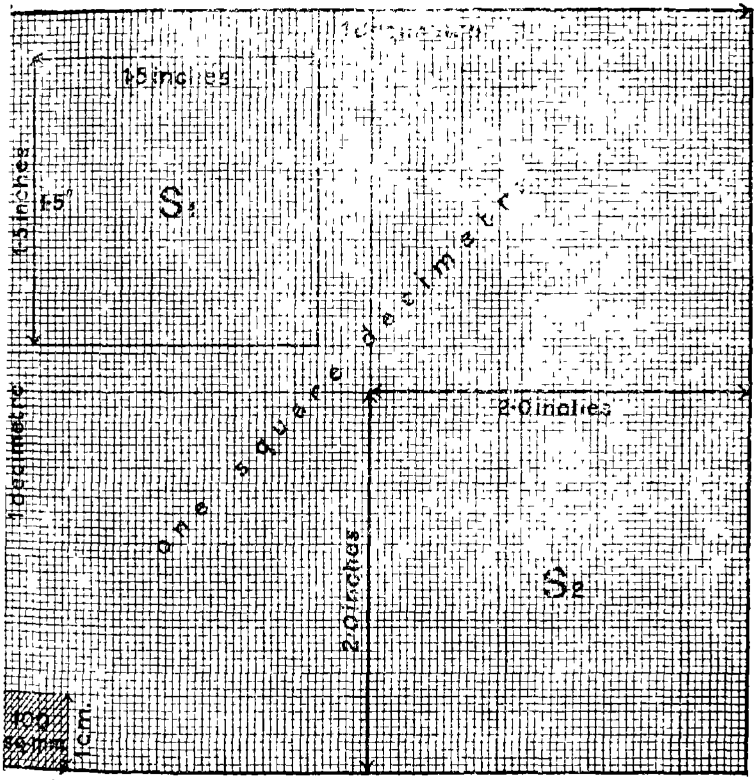
Fig. 16.

Ex. i. In Fig. 16 three figures have been drawn, and it is evident from inspection of *A* and *B* that the area of each is obtained by counting the small squares. The area of *C* may be obtained by counting only the small squares

(enclosed by the lines of the triangle) which are greater than half-squares: these are indicated by dots placed in the squares which must be counted; the rest may be neglected.

16. Practical Exercises on Area.

1. On millimetre squared paper draw two squares with sides 1 cm. and 1 dm. respectively. **Find** from these the **number of sq. mm. in 1 sq. cm.** and the **number** of **sq. cm.** in **1 sq. dm.** (see Fig. 17).

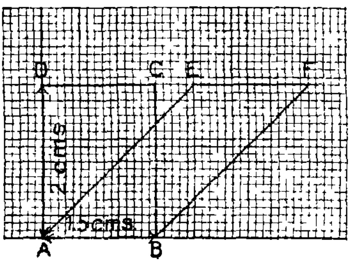


1 sq. cm. = 100 sq. mm. 1 sq. dm. = 100 sq. cm. = 10,000 sq. mm.

Fig. 17.

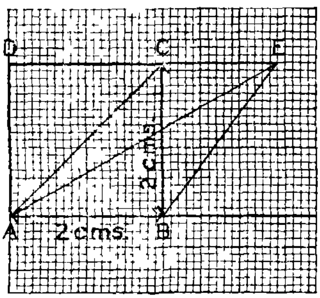
2. Draw two squares with sides 1·5″ and 2″ respectively. Find the number of sq. mm. in each and obtain two values for the **number of sq. mm. in 1 sq. in.** Take the mean of these (see Fig. 17).

3. **To find the area of a parallelogram.** Draw a rectangle *ABCD* (Fig. 18) with base $AB = 1 \cdot 5$ cm., height $BC = 2$ cm. On *DC* produced take point *E* 5 mm. from *C*, and a point *F* 20 mm. from *C*. Join *AE* and *BF*. Find the area of the parm. (= parallelogram) *AEFB* by counting squares.



Area of parallelogram =

Fig. 18.



Area of triangle =

Fig. 19.

What is the difference between the area of the parm. and the area of *ABCD*, the rectangle on the same base and of same perpendicular height? State rule for finding area of any parm.

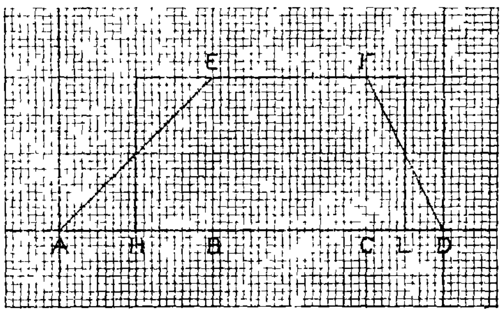
Area of parallelogram =

4. **To find the area of a triangle.** Draw a line *AB* 2 cm. long (Fig. 19). From *B* draw a perpendicular *BC* 2 cm. long. Join *AC*. Find area of $\triangle ABC$ by counting squares. Draw *AD* parallel to *BC*, and *CD* parallel to *AB*. Find area of rectangle *ABCD*.

What relation is there between area of rectangle and area of triangle on same base and of same perpendicular height? Take any point E on DC produced. Join AE, BE and find area of $\triangle AEB$. What is the difference between area of $\triangle AEB$ and of $\triangle ABC$? Give the rule for finding the area of any triangle.

Area of triangle =

5. **To find the area of a trapezium.** Draw a line $ABCD$ (Fig. 20), making $AB = 2$ cm., $BC = 2$ cm., $CD = 1$ cm. Mark points E and F 2 cm. vertically above B and C respectively. Join $AEFD$. Find area of trapezium[1] $AEFD$ so formed by counting squares. Bisect AB at H, CD at L and construct



Area of trapezium =

Fig. 20.

rectangle on base HL of same height as trapezium. State (*a*) difference between the area of this rectangle and the area of the trapezium, (*b*) difference between HL and the mean of EF and AD, (*c*) rule by which area of any trapezium may be found.

[1] A *trapezium* is a 4-sided figure with two of its sides parallel.

EXAMPLES II A.

RECTANGLE, PARALLELOGRAM, TRIANGLE AND TRAPEZIUM.

1. How many sq. cm. are there in (*a*) 1 sq. m., (*b*) 1 sq. mm., (*c*) 1 sq. dm., (*d*) 1 sq. Dm.?

2. Express in sq. m. (*a*) 25 sq. dm., (*b*) 2·5 sq. Dm., (*c*) 625 sq. cm., (*d*) 100 sq. mm.

3. Add 0·005 sq. m., 0·5 sq. dm., 500 sq. mm., 5 sq. Dm.
Give answer in sq. cm.

4. If 1 inch = 2·54 cm., how many sq. cm. are there in 1 sq. in.?

5. How many sq. yds. are equal to 1 sq. m.?

6. Find the area in sq. cm. and sq. m. of a rectangle 3·6 dm. by 270 mm.

7. How many visiting cards 7·6 cm. by 38 mm. can be cut from a square piece of cardboard the side of which is 3·8 dm. long?

8. What length of carpet 2′ 3″ wide will be required to cover a floor 24 sq yds. in area?

9. What is the difference between 7 sq. cm. and a 7 cm. square.

10. A sheet of note-paper is 4·41″ or 11·2 cm. wide and 7″ or 17·85 cm long. Calculate its area in sq. in. and sq. cm. and find number of sq. cm. in 1 sq. in. (to 2 decimal places).

11. Find the area in sq. cm. of a $\triangle$ (*a*) with base 8·2 cm. and perpendicular height 35 mm., (*b*) with base 5″ and height 10 cm., (*c*) with height x cm. and base = $\frac{1}{2}$ height.

12. Find the height of a $\triangle$ with (*a*) base 40 mm., area 7 sq. cm., (*b*) base 70 mm., area 0·1925 sq. dm., (*c*) base = twice height, area = 144 sq. in.

13. The area of a parallelogram, whose base is one-third the height, is 75 sq. cm. Calculate height and base.

14. A trapezium has two parallel sides 14 cm. and 30 cm long and the perpendicular distance between these is 8 cm. Find area of figure.

15. If the pressure of the atmosphere is 15 lb. per sq. in., what is the pressure on 1 sq. cm.?

17. Practical Exercises on Area of Circle, Cylinder, Cone and Sphere.

1. **To find the area of a circle.** Draw on inch squared paper a circle of diameter 2″ (Fig. 21). Find area by counting squares. Divide this by the area of the square on radius of

circle. Repeat with a circle of a different diameter. Give rule for finding area of any circle.

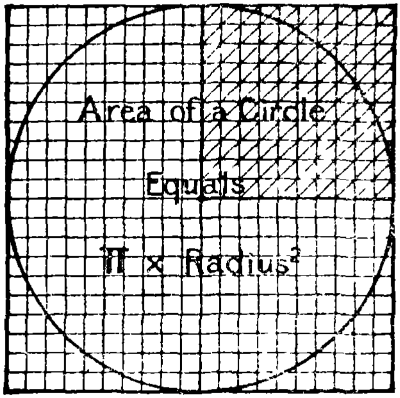


Fig. 21.

Area of a circle =

2. Draw a circle of diameter 6″. Divide into 16 *equal* sections by lines through the centre. Cut these out and fit them together to form a parallelogram (Fig 22). Measure base and

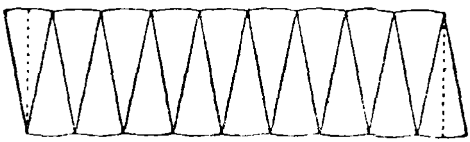


Fig. 22

height of this parallelogram and compare them with the radius and circumference of the circle. Suggest formula for area of a circle.

Area of $\odot$ = Sum of areas of $\triangle$s

$$= \frac{\text{Sum of bases} \times \text{height}}{2}$$

$$= \frac{\text{circumference of } \odot \times \text{radius}}{2} = \frac{2\pi r \times r}{2} = \pi r^2.$$

3. **To find the area of the curved surface of a cylinder.** Cut out a piece of paper which will just wrap round a cylinder. What measurements of the cylinder will give the length and breadth of this paper?

Deduce the rule for finding the area of the curved surface of any cylinder.

Area of curved surface =

4. **To find the area of the curved surface of a cone.** Cut out a piece of paper that will just cover the curved surface. Divide it into $\triangle$s by lines drawn from vertex to points equally distant along base. How could the area of the paper now be found? What measurements on the cone are equal to the height and the sum of the bases of these $\triangle$s. State rule for finding the area of the curved surface of any cone (**= πr × slant height**).

5. **The area of the surface of a sphere**

= the area of the curved surface of the circumscribing cylinder

$= 2\pi r \times 2r = \mathbf{4\pi r^2}$.

EXAMPLES II B.

AREA OF CIRCLE, CYLINDER, CONE AND SPHERE.

1. Find the area of a circle of radius (*a*) 4 cm., (*b*) 1·4″, (*c*) 0·56 dm.

2. Calculate the radius of a circle of area (*a*) 154 sq. cm., (*b*) 314$\frac{2}{7}$ sq. in., (*c*) 13·86 sq. dm.

3. What is the area of a circle the circumference of which is (*a*) 44″, (*b*) 26·4 cm., (*c*) 22 cm.?

4. Find the area of the curved surface of a cylinder of (*a*) height 6·3 cm., diameter 4·4 cm., (*b*) height 1′ 2″, diameter 10″, (*c*) height 16″, diameter 8·4″.

5. What is the height of a cylinder if (*a*) diameter = 7 cm. and area of curved surface = 176 sq. cm., (*b*) diameter = 10 cm., area = 704 sq. dm., (*c*) radius = 31·5 mm., area = 158·4 sq. cm.?

6. Calculate area of curved surface of the following cones:

(*a*) Slant height 7″; radius of base 5″.

(*b*) ,, ,, 6 cm.; diameter of base 5 cm.

(*c*) ,, ,, 4·5 cm.; circumference of base 10 cm.

7. How much leather is required to cover

(*a*) a cricket ball of diameter 2·7″,

(*b*) a football of diameter 9″?

8. Find the area of a circular racing track if diameter of inner circle is 140 yds. and width of track $3\frac{1}{2}$ yds.

9. The pressure of the steam inside a boiler is 80 lb. per sq. in. What is the pressure on a circular valve of diameter $3\frac{1}{2}$″?

10. How much canvas will be required for a round tent of diameter 5 yds. 1 ft. and height 8 ft., with a conical top 2 yds. high?

Summary of Formulae.

Area of parallelogram = base × perpendicular height.

Area of triangle $= \dfrac{\text{base} \times \text{perpendicular height}}{2}$.

Area of trapezium = half sum of parallel sides × perpendicular height

Area of circle $= \pi r^2$.

Area of curved surface of cylinder $= 2\pi rl$.

Area of surface of sphere $= 4\pi r^2$.

CHAPTER III.

MEASUREMENT OF VOLUME.

18. Volume has three dimensions.

Volume measures the space a body occupies. You are given eight one-inch cubes. Note that each face is a square. The *volume* of each is called *one cubic inch*. Their total volume is the same whether you pile them (*a*) one above the other, or (*b*) build them to form a cube each edge of which measures 2 inches (Fig. 23 *a*).

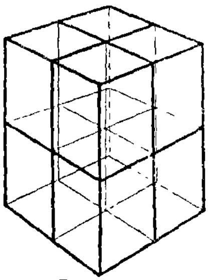


Fig. 23 *a*.

The area of the base × height gives the volume.

In (*a*) 1 inch × 1 inch × 8 inches
= 8 cubic inches;

in (*b*) 2 inches × 2 inches × 2 inches = 8 cubic inches.

The volume of a solid depends therefore on *three dimensions—length, breadth* and *height.*

***Ex. Find the number of cubic centimetres in one cubic inch.**

Measure the edges of one of the blocks in centimetres.

$$\text{Length} \times \text{breadth} \times \text{height} = \text{volume},$$

$$1 \text{ inch} \times 1 \text{ inch} \times 1 \text{ inch} = 1^3 = 1 \text{ cubic inch},$$

$$2{\cdot}54 \text{ cm.} \times 2{\cdot}54 \text{ cm.} \times 2{\cdot}54 \text{ cm.} = (2{\cdot}54 \text{ cm.})^3 = 16{\cdot}39 \text{ cu. cms. (c.c.)}.$$

Metric System.

A cubic decimetre = 10 x 10 x 10 c.c.
= 10^3 c.c. = 1000 c.c.

This volume is called a **litre.**

A Litre = 1000 c.c. = $1\frac{3}{4}$ pints (nearly).

1 cubic metre = 100^3 c.c. = 1,000,000 c.c. = 1000 litres.

British System.

1 cubic foot = 12^3 cu. ins. = 1728 cu. ins.

1 cubic yard = 3^3 cu. ft. = 27 cu. ft.

1·31 cubic yards (approx.) = 1 cubic metre.

19. To find the Volume of a Prism.

A right-prism is a solid enclosed by two parallel planes, connected by rectangular sides.

We can build prisms with the 8 inch-cubes, and we have already seen that **the volume is obtained by multiplying the area of the base by the height.**

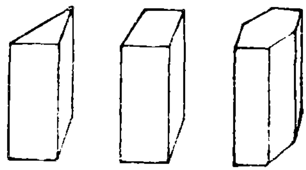


Fig. 23 *b*.

In Fig. 23 *b* a triangular, a rectangular and a hexagonal prism are shown. Inspection tells us that by varying the height we can vary the volume of the prism, *i.e.* the volume is proportional to the height.

The cylinder is a right-prism with an infinite number of sides. Its base is a circle (Fig. 24).

The vol. of a cylinder = area of the base x height

$= \pi r^2 h.$

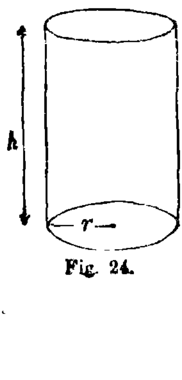


Fig. 24.

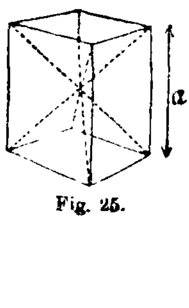


Fig. 25.

20. To find the Volume of a Pyramid.

If we join the opposite corners of a cube of edge a, we shall divide it into six equal pyramids (Fig. 25), each of volume $\frac{a^3}{6}$.

The base of each pyramid $= a^2$.

The height of " " $= \frac{a}{2}$.

The volume of " " $= a^2 \times \frac{a}{2} \times \frac{1}{3} = \frac{a^3}{6}$.

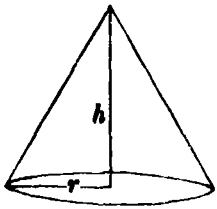


Fig. 26.

The volume of a pyramid $= \frac{1}{3}$ base $\times$ perpendicular height.

A pyramid may have a base of 3, 4, 5 or more sides.

To find the Volume of a Cone.

A cone is a pyramid whose base is a circle (Fig. 26).

The vol. of a cone $= \frac{1}{3}\pi r^2 h$.

21. To find the Volume of a Sphere.

We imagine the surface of a sphere to be divided into an infinite number of small figures, each of which is practically a plane surface (Fig. 27). We can also imagine the corners of these figures joined to the centre of the sphere, thus making an infinite number of pyramids with their tops (or apices) at the centre, all having the same height, r (the radius of the sphere). The sum of these small pyramids is the volume of the sphere and their total base area is the whole surface of the sphere.

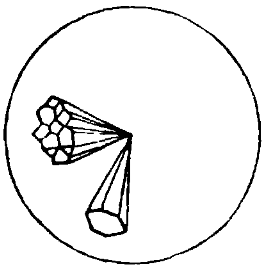


Fig. 27.

The vol. of the sphere

$= \frac{1}{3}$ sum of bases of the pyramids $\times$ height

$= \frac{1}{3}$ surface of the sphere $\times$ radius

$= \frac{1}{3} \times 4\pi r^2 \times r = \frac{4}{3}\pi r^3$.

***Practical Exercise.** Find the volumes of the given (*a*) cylinder and (*b*) sphere both in cu. inches and c.c. and calculate the number of c.c. in one cu. inch from each set of results.

22. The use of Measuring Flasks, Graduated Cylinders, Pipettes and Burettes[1].

(*a*) **Flasks** of sizes 50 c.c., 100 c.c., 250 c.c. and 1000 c.c. (litre) are made with narrow necks on which a graduation mark is etched according to the capacity (Fig. 28 *a*).

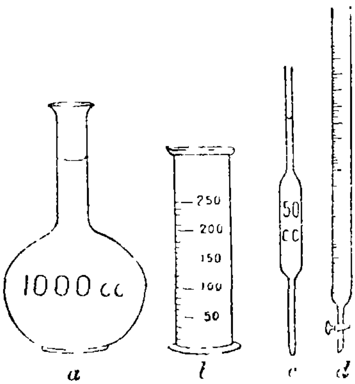


Fig. 28.

(*b*) **Cylinders** (Fig. 28 *b*). Care must be taken to note the number of c.c. corresponding to the graduation and to see that the reading is taken from the bottom of the *meniscus* (or curved surface of the liquid). The volume of a small solid may be obtained by lowering it carefully into water in the cylinder, and noting the rise of the surface.

(*c*) **Pipettes** (Fig. 28 *c*) of definite volume are filled by sucking up liquid to a mark on the stem, the end is then closed with the finger by which the outflow of liquid may be controlled.

(*d*) **Burettes** (Fig. 28 *d*) are long narrow tubes, generally

[1] To guard against *Parallax Error*, read *Exp. Science* II. *Chemistry*, § 8.

graduated to $\frac{1}{10}$ c.c., and having a delivery tap by which small quantities of liquid may be run out with great accuracy.

23. To find the Volume of an Irregular Solid.

Mention has already been made (§ 22 *b*) of a method of finding the volume of a solid by noting the rise of the surface of the water in a graduated cylinder when the solid is carefully lowered into it. The method shown in Fig. 29 is on the same principle but the water displaced overflows and is caught and

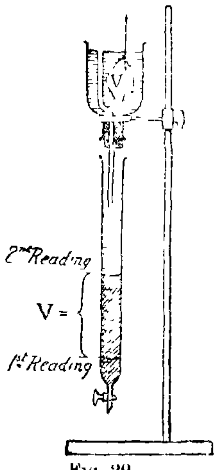


Fig. 29.

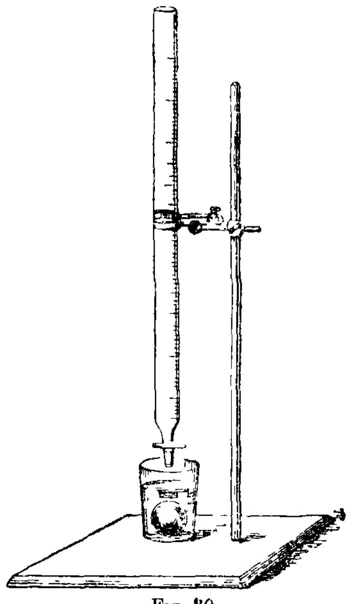


Fig. 30.

measured in a burette. Water is poured into the upper cistern until some of it runs through the internal tube into the burette. When the water has ceased to drop the burette reading is taken (say 47·6 c.c.). The solid, the volume (V) of which is

required, is then carefully lowered into the cistern and the overflow is noted by reading the new level in the burette (say 20·2 c.c.). The difference (47·6 − 20·2 = 27·4 c.c.) gives the volume, V, required.

***Practical Exercises.** 1. A cylinder and a cone of equal bases and equal heights are given; prove "by displacement" that the volume of the cone = $\frac{1}{3}$ volume of the cylinder.

*2. Having measured the diameter of the given sphere with the calipers, devise a displacement method (see Fig. 30) for finding its volume, and verify the formula $V = \frac{4}{3}\pi r^3$.

*3. Two spheres are provided, the diameters of the one being double that of the other; prove that their volumes are as $1 : 2^3$.

Examples III a (Volume).

1. How many cubic centimetres are there in (*a*) 1 c.m., (*b*) 1 c.mm., (*c*) 1 c.dm.?

2. Express 12345 c.c. in (*a*) c.m., (*b*) c.mm., (*c*) c.Dm.

3. How many cubic metres are there in (*a*) 0·0067 c.Dm., (*b*) 1231 c.c., (*c*) 567 c.dm., (*d*) 6800 c.mm.?

4. Add together 0·005 c.m., 500 c.c., 5000 c.mm. Give answer in c.dm.

5. A box is 3 dm. long, 20 cm. wide and 80 mm. deep. Calculate its volume in c.c. and c.dm.

6. If there are 25·4 mm. in 1 inch find to three decimal places the number of c.mm. in 1 cubic inch.

7. How many cubic decimetres are there in 1 cubic foot?

8. What are the volumes of rectangular blocks of wood of the following dimensions: (*a*) 8″ by 5·2″ by 0·6″, (*b*) 0·178 m. by 1·2 dm. by 9 mm.?

9. Calculate the height of a rectangular box of
   (*a*) volume = 231 c. in., area of base = 55 sq. in.,
   (*b*) volume = 0·84 c.m., length = 80 cm., breadth = 7 dm.

10. Find the volume in cubic centimetres of a triangular prism 5·5 cm. high
    (*a*) if base of triangle = 3 cm., height = 22 mm.,
    (*b*) if base of triangle = 4·2 cm., height = 1 inch.

11. Express in c.c. (*a*) 0·011 litre, (*b*) 500 centilitres, (*c*) 54 millilitres.

12. Add together 0·25 litre, 500 millilitres, 0·5 decalitre, 60 decilitres Give answer in cubic centimetres.

13. What difference is there in c.c. between ·006 c.dm. and ·06 litre?

14. If a tank measures 8 dm. by 50 cm. by 400 mm., how many litres of water will it hold?

15. A tank is 1·8 m. long and 90 cm. wide. A decalitre flask can be filled 81 times from the tank. Find the depth.

16. How many days will a hectolitre of oil last for 8 lamps each burning ½ pint a day. (1 litre = 1·76 pint.)

EXAMPLES III B (VOLUME OF CYLINDER, CONE, SPHERE AND PYRAMID).

1. If the length of a burette from 0 to the 10 c.c. mark is 9 cm., what is the area of cross-section in square millimetres?

2. How many metres of wire of 0·4 sq. mm. cross-section can be drawn from 5 c.c. of metal?

3. The internal diameter of a piece of glass tubing is 2 mm. What length of the tubing will hold 0 55 c.c. of water?

4. The volume of a piece of wire 4000 cm. long is $31\frac{3}{7}$ c.c. Calculate the diameter of the wire.

5. The circumference of a cylinder is 44 cm.: its volume 1540 c.c. Find its height and diameter.

6. Calculate the volume of a cone (*a*) 6 cm. high, of radius 5 cm., (*b*) 10 cm. high, of diameter 4·2 cm.

7. A cone 7 cm. high and of diameter 3 cm. just slips into a graduated cylinder. How much water must be poured in to raise level to apex of cone?

8. Find the height of a cone which has the same base and volume as a cylinder 4 cm. high, of radius 2·1 cm.

9. How many cubic feet of gas will be contained by a balloon 42 ft. in diameter?

10. Calculate the volume of a sphere whose surface is 616 sq. cm.

11. A lead bullet of diameter 6 mm. is beaten out into a circular disc of diameter 10 mm. What will be the thickness of the disc?

12. Find the ratio of the volumes of a cylinder, cone, and hemisphere of the same base and height.

13. The largest gasometer in the world is 300 ft. in diameter and 180 ft. high. How much gas can it hold?

14. A square pyramid is 8 inches long and 8 inches high. Find its volume.

15. A square pyramid is 12 cm. long and its slant height is 10 cm. Calculate its volume. (The *slant height* is the short distance, measured on one of the faces, from the apex to the side of the base.)

CHAPTER IV.

MEASUREMENT OF MASS AND WEIGHT.

24. Mass.

The *mass* of a body is the quantity of *matter* which it contains.

Matter is difficult to define; all objects around us—earth, water, air, wood—are different forms of matter.

(1) *Matter occupies space.*

(2) *Matter is inert,* i.e. it has no power in itself to move, if it is at rest, or, if it is in motion, to cease moving.

(3) *Every* little piece (*particle*) of matter in the universe pulls or *attracts every other particle* to it. The shorter the distance between two bodies and the greater their masses, the greater is their attraction to each other.

Weight.

The *pull* or *attraction* between the Earth and a body, such as a stone, we call the *weight* of the stone. The pull may be measured by a **spring balance.** If we were to hang a stone to an accurate spring balance at the Equator, we could record the pull of the Earth or the *weight* of the body at the Equator. As we move N. or S. towards the poles, the spring balance would record a greater pull. The stone is nearer the Earth's centre as we move N. or S. from the Equator, the Equatorial radius being greater than the Polar radius; hence the Earth's pull on the stone or the *weight* of the stone is greater at the Poles, although its *mass* remains the same. Weighed at high altitudes in a balloon, the stone's *weight* would be less.

25. How to compare Masses.

(1) A **"pair of scales,"** with equal arms (§ 27), is used to **balance** one mass with another. A pound mass of lead and a pound mass of nails once balanced at any place, in London for instance, will always balance anywhere, because the pull of the Earth on each mass in the balance pans is the same at one place.

(2) **By the spring-balance.** But if the lead, hung on a spring-balance, is carried to the Poles, and if the nails, also suspended from a spring-balance, are carried to the Equator, the lead *weighs* more and the nails weigh less, as explained above. If then **weight** as recorded by a spring-balance is to be taken as a **measure of mass,** the *place of observation must not be changed,* **weight being then proportional to mass.**

26. The British Unit of Mass—the Pound.

We can compare masses by *balancing* them against the *same mass.* The particular British unit chosen is the mass of a piece of Platinum kept at the Board of Trade Offices, and is called the **imperial standard pound.**

One pound (Avoirdupois) = 16 ounces = 7000 grains.

The Metric Unit of Mass—the Kilogram is the amount of matter in a piece of Platinum kept by the French Government. The mass of this **standard kilogram** (1000 grams) is equal to **the mass of a litre (1000 c.c.) of pure water at 4° C** In the Metric System there is therefore a connection between the unit of *mass* and the unit of *volume*

1 kilogram = 1000 grams = 2·2 lbs. Av.[1]
1 hectogram = 100 grams.
1 dekagram = 10 grams.
1 gram = mass of 1 c.c. of water at 4° C.
= 15·432 grains.
0·1 gram = 1 decigram.
0·01 gram = 1 centigram.
0·001 gram = 1 milligram.

1 ounce (avoirdupois) = 28·35 **grams.**

[1] More accurately 2·205 Avoirdupois.

27. The Balance consists of a rigid beam (Fig. 31), supported at its centre by a knife edge C which rests on an agate plane attached to a vertical support. Two pans of equal weight are hung on knife-edges (A and B) at the ends of the beam, the essential feature of the balance being that the distances of the outer knife-edges from the central knife-edge are equal, *i.e. the "arm" AC = the "arm" CB.* The knife-edges

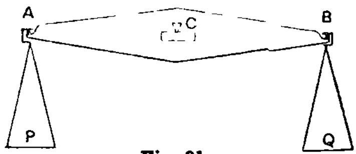


Fig. 31.

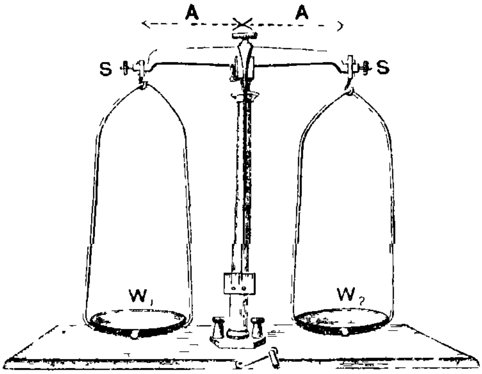


Fig. 32.

and agate planes on which they rest are easily damaged. Various mechanical contrivances are attached for levelling and for lifting the knife-edges from the agate planes. Carefully examine the balance which you are using. Notice that the amount of swing is shown by a pointer attached to the beam. To obtain an accurate weighing, it is of the utmost importance that the pointer should swing evenly to equal distances on each side of the central line of the scale attached to the support

Fig. 32 shows the essentials of a simple balance—*equal arms* AA, a *lever* to raise the plane on which the central knife-edge rests, and *nuts* or *screws* SS adjusted to obtain an even swing

before weighing: weights W_1 W_2 placed in the pans will have equal masses when the balance again *swings evenly*.

28. Rules to be observed in Weighing.

1. See that the weights are correct before and after use and that they are in their right places in the box. The weights should be multiples and submultiples of 5, 2, 2, 1 grams or 5, 2, 1, 1.

2. Never put anything on, or remove anything from, the pans when the balance is free to swing, and never leave it swinging.

3. First see that the balance is correctly adjusted and then always put the substance to be weighed in the left-hand pan.

4. Never touch the balance or the weights with your fingers if you can possibly help it.

5. Always begin with a weight that is too great. Remove this weight and continue to add weights in decreasing order of magnitude.

6. Never weigh anything hot, or wet, or likely to stick to the pan.

7. Raise the lever gently with the left hand, and only turn it completely over when a balance is nearly obtained.

8. Count the weights *both* by the number on the pan *and* by the vacant places in the box.

29. *Practical Exercises in Weighing.

(1) Find the number of grams in one ounce (Avoirdupois) by weighing the 1 oz. weight provided. Compare your result with the value given in § 26.

(2) **To test the accuracy of a pipette.** Cool some water to 4° C. by adding ice to tap water. Weigh an empty beaker (capacity about 100 c.c.). Fill the pipette (by suction) to the mark with water at 4° C., and let it *drain* into the weighed beaker. (N.B. The water should not be blown out of the pipette.) Weigh the beaker and water: subtract the weight of the empty beaker—the difference in grams should equal the number of c.c. marked on the pipette.

(3) **Find the weight of 1 c.c. of methylated spirit.** Repeat, filling the same pipette with methylated spirit. Knowing the volume of the pipette in c.c., calculate the weight of 1 c.c. of spirit.

(4) **To measure area by weighing.** You are given a piece of cardboard of uniform thickness on one side of which is pasted *centimetre-squared paper.*

(*a*) Accurately cut out a square of 100 sq. cm. area (side = 10 cm.) and weigh this cardboard square. Calculate

Weight of 1 square centimetre

$$= \frac{\text{Weight of square (100 sq. cm.)}}{100}.$$

(*b*) Draw and cut out accurately a circle, radius 5 cm. Find its area in sq. cm. by counting the squares *or* by calculation. Weigh the cardboard circle and divide the weight by the wt. of 1 sq. cm. of cardboard found in (*a*) and thus obtain the ***area*** indirectly *by weighing.*

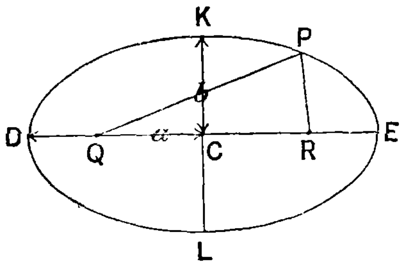


Fig. 33.

(*c*) Cut out any irregular figure and ***weigh*** **it. Find the area** (1) by counting squares, (2) by dividing the weight of the cardboard figure by the weight of 1 sq. cm. of cardboard.

(*d*) On the "squared" cardboard, describe an **ellipse** (see Fig. 33) by firmly fixing two pins at points Q and R about 5 cm. apart: over the pins put a loop of cotton (about 12 cm. long): place a pencil in the loop and, keeping the cotton tight, describe the ellipse with the point P. Produce QR to the curve at D and E. Bisect QR at right angles by the line KCL. DE and KL are the axes of the ellipse and equal $2a$ and $2b$ respectively. Cut out the ellipse and show (1) by weighing, (2) by counting squares, that the **area** $= \pi ab$, where a and b are the semi-axes.

30. Calibration of a Spiral Spring Balance (Dynamometer).

Fig. 34 *a* represents a simple form of spring-balance and Fig. 34 *b* a *dynamometer* or "force measurer" constructed on the same principle. A triangular groove cut in a block of wood

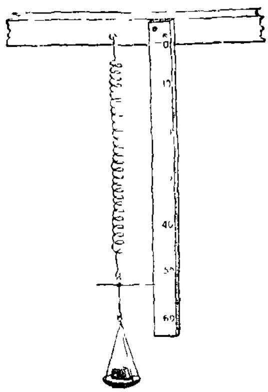


Fig. 34 *a*.

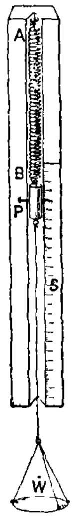


Fig. 34 *b*.

Weights added, grams	Scale Readings, cm.	Extension, cm.	Extension per 10 grams
0	1·30	0·00	0·00
10	3·91	2·61	2·61
20	6·51	5·21	2·60
30	9·13	7·83	2·62
40	11·73	10·43	2·60
50	14·34	13·04	2·61
... &c.	...	...	...

contains a bell spring AB (or a rubber cord) attached at A by a hook. A short pencil pierced by a pin P (the *pointer*) has one end connected to the spring and the other to a thread which carries the weight-pan (W). The extension of the spring is read from the pointer on the scale S.

Graph of the Extension.

***Ex. i. Take readings on the scale when weights are added to the pan** as in the table on p. 40.

N.B. Take care not to strain the spring.

Gum a label on the pencil giving the average extension per 10 grams = 2·61 cm. The spring and pencil may be detached and used on any similar groove and scale.

To plot the curve. On squared paper draw the axes of X and Y to represent *weights added* (column 1) and *extension* (column 3) respectively (Fig. 35 [see also § 13]).

Weighing by use of dynamometer and graph.

***Ex. ii. To find the number of grams in 2 ozs.** Place a two-ounce weight in the pan. Note that the scale reading is 16·1 and the *extension* of the spring is *14·8 cm.* Referring to the graph $\frac{\text{extension}}{\text{weights added}}$ in **Fig. 35,** we find that the *abscissa* (on the axis of X) corresponding to the *ordinate* 14·8 cm. (on the axis of Y) is 56·5 grams (see point P in Fig. 35). Test the accuracy of your result by using the ordinary balance (scales), thus proving that the **weight of a body is proportional to its mass** [§ 25 (2)].

***Ex. iii.** Weigh a *one-ounce weight* and other suitable objects which will not strain the dynamometer.

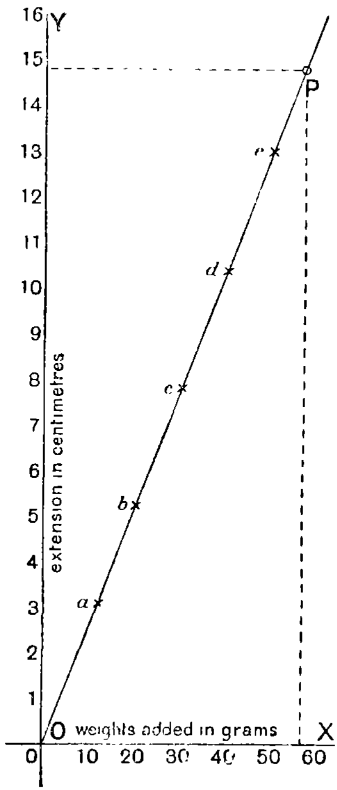


Fig. 35.

EXAMPLES IV (WEIGHT).

1. How many milligrams are there in (*a*) 1 Dgm., (*b*) 1 cgm., (*c*) 1 gm., (*d*) 0·1 gm.?

2. How many grams are there in (*a*) 500 mgm., (*b*) 52 dgm., (*c*) 105 cgm., (*d*) 0·006 Kgm.?

3. Express 12345 cgm. as (*a*) Kgm., (*b*) gm., (*c*) Hgm., (*d*) mgm.

4. What is the sum of

   (*a*) 1 Kgm. + 2 Hgm. + 3 dgm. + 4 mgm.,

   (*b*) 2 dgm. + 505 mgm. + 2 gm. + 100 mgm.?

5. Subtract

   (*a*) 505 mgm. from 12·545 gm., (*b*) 1 dgm. from 1 Dgm.,

   (*c*) 10 gm. from 0·01 Kgm., (*d*) 16 gm. from 0·175 Hgm.

6. Add 1·607 Kgm., 546 dgm., 243 cgm., 505 mgm.

   Give answer in (*a*) gm., (*b*) Kgm.

7. If 4 dm. of wire weigh 16 dgm. how many grams will 1 metre of the wire weigh? What length of wire would be required to make a cgm. weight?

8. 14 cm. of wire weigh 8 dgm. How many mm. will weigh 20 mgm.?

9. If 1 oz. (avoir.) = 28·35 gm., calculate the number of (*a*) grams in 1 lb., (*b*) lbs. in 1 Kgm.

10. If a man weigh 70 Kgm., what is his weight in lbs.?

11. A flask weighs 70 gm. when empty and 209 gm. when full of water. What is its capacity?

12. A 50 c.c. bottle weighs 70 gm. when full of water. Find weight of bottle.

13. If 11 c.c. of a solution contain 3·08 gm. of salt, how much salt will there be in a decilitre?

14. A solution is to be made containing 1 drachm of salt in one pint of water. How many grams per litre will this be?

15. A tank holds 63·8 lbs. of water. What is its volume in litres (1 Kgm. = 2·2 lbs.)?

16. If a gallon of water weighs 10 lbs. and 1 oz. = 28·35 gm. find the number of litres in a gallon.

17. An ebony cylinder placed in the scale-pan of a dynamometer causes the pointer to move to the mark 4·4 cm. From Fig. 35 find the weight of the cylinder.

18. What reading will be registered by the same dynamometer if an article weighing 36 gm. be placed in the scale-pan?

Miscellaneous Questions on Measurement (Chapters I—IV).

1. If a cyclist travels 18 Km. per hour what is his speed in centimetres per second?

2. 11 sq. cm. of a sheet of Aluminium weigh 1·25 gm. What area would be needed to make a 1 gm. weight?

3. From a circular disc of cardboard of radius 10 cm. a triangle of base 8 cm. and height 3·5 cm. is cut out. Find the area of the remainder.

4. A steam roller $4\frac{1}{2}$ ft. wide has a diameter of 3 ft. How many times will the roller revolve in traversing a road $314\frac{2}{7}$ yds. long? What area of the road will have been rolled while this distance is travelled?

5. What is the depth of a tank which will hold 105 cu. ft. of water if the length and breadth be 10 ft. and $3\frac{1}{2}$ ft. *respectively?*

---

6. If 1 metre = 39·37 inches, how many kilometres are there in 1 mile?

7. What is the area of a pond, the circumference of which measures 220 yds.?

8. If 2·5 metres of wire weigh 9·5 decigrams, how many milligrams will 5 cm. of wire weigh?

9. A rectangular piece of Aluminium is 8·5 cm. long and 3 cm. wide: its weight is 2·73 gm. If 1 c.c. of Aluminium weighs 2·6 gm., find the thickness of the metal.

10. How many gallons of water will a tank contain if its dimensions are 3 ft. × 1 ft. × 8 in. ($6\frac{1}{4}$ gallons = 1 cu. ft.).

---

11. The diameter of a penny is $\frac{1}{10}$ ft. How many pence put edge to edge will reach 3048 mm.?

12. If 8 sq. cm. of a sheet of metal weigh 24 gm., what is the area of an irregular figure weighing 14·4 gm. cut from the same sheet?

13. Find the area of the surface of a square pyramid if the slant height be 8 inches and the side of the base 5 inches.

14. If the pitch of a screw gauge is 0·5 mm. and the edge of the collar is divided into 50 equal parts, how many complete turns of the collar will be needed to open the jaws 2·87 mm.? What number on the collar will be opposite the zero line?

15. What weight of water will there be in a tank 6 ft. × 4 ft. × $2\frac{1}{2}$ ft. if 1 gallon of water weighs 10 lbs.?

---

16. Calculate in kilometres per hour the speed of a train travelling 60 miles per hour.

17. A rectangular piece of copper foil, length 8 cm., width 3 cm., weighs 3·2 gm. An irregular figure weighing 2·52 gm. is cut from the same foil. What is its area?

18. Find the area of a trapezium the parallel sides being 8″ and 10″ long and the perpendicular distance between them 5″.

19. A copper triangle of base 10 cm. and height 10 cm. weighs 5·34 gm. If 1 c.c. of copper weighs 8·9 gm., calculate the thickness of the metal.

20. Find the height of a cylinder (diameter 2 ft.) which will hold 50 gallons of water.

---

21. A body falling freely increases its speed in every second by 32·2 ft per second: calculate this acceleration in centimetres per second.

22. Find the total area of a hot-water cylinder of height 3 ft. 7 in and diameter 1 ft. 2 in.

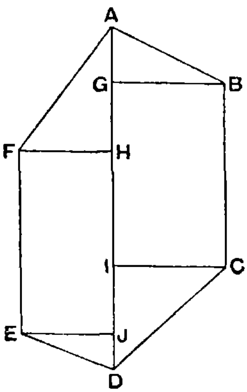


Fig. 36.

23. In measuring a field the surveyor takes a straight line across the longest part of it and measures from this line perpendicular distances to the various corners. The extremities of these lines when joined give the plan *ABCDEF*.

$AG = 100$ m., $GH = 120$ m.,
GB and $IC = 200$ m., $HI = 200$ m.,
$IJ = 120$ m., $DJ = 60$ m.,
EJ and $FH = 160$ m.

Calculate area of field.

24. Twenty-seven marbles of diameter 2 cm. just fit into a cubical box. What is the length of the cube? How many c.c. of water would be needed to fill up the space in the box between the marbles?

25. A conical tent has a diameter of 12 yds. and vertical height 7 yds. What volume of air will it contain?

---

26. Five marbles of radius $\frac{3}{4}$ inch just fit into a cylindrical box. Calculate the height and diameter of the box.

27. Calculate the volume of a pyramid if vertical height be 6 ft. and rectangular base 15 ft. by 10 ft.

28. A cylindrical boiler is fitted with hemispherical ends. If the length of the cylindrical part is 10 ft. and its diameter 4 ft., find to the nearest penny the cost of painting the outside of the boiler at 3*d.* per sq. ft. (O. J.)

29. The internal dimensions of a closed cylindrical tank are length 10 ft. 3 in.: diameter 4 ft. 2 in. If the metal of which it is made is 1 inch thick, find in cubic feet the volume of the metal. (O. J.)

30. From the following notes draw a plan of the field to scale and find the acreage of the field. (The field is supposed to be enclosed by straight hedges and have 5 corners.)

A chain (22 yards) = 100 links.

An acre = 4840 square yards.

	Links	
	to *D*	
	1000	
	800	420 to *C*
480 to *E*	540	
	100	320 to *B*
	from *A*	

31. Make a summary of all the formulae you have used in finding lengths, areas and volumes.

CHAPTER V.

MEASUREMENT OF DENSITY BY SIMPLE METHODS.

31. Density: the Mass per unit Volume.

A cube of *lead* (b) and a cube of cork (a) of exactly *the same size* were weighed (Fig. 37). The cube of *lead* weighed one pound and the equal-sized cube of cork weighed 0·28 of an ounce. By dividing 1 lb. by 0·28 oz., we find that the leaden cube was 57 times heavier than the cork cube of equal size.

Fig. 37.

In Fig. 37 we have shown the relative sizes of cubes of cork (A) and of lead (b), *each of which weigh one pound.* Clearly it is not correct to say of the two cubes A and b that "the lead is 57 times heavier than the cork," for each cube (A and b) weighs one pound. It is therefore not correct to say that "lead is 57 times heavier than cork" unless we assume that we refer to *equal volumes* of the substances. The adjective we use when we wish to compare weights of equal volumes of substances is the word *dense* and the corresponding noun is the word **density**. It is correct to say that

(1) "lead is 57 times as dense as cork,"

or (2) the density of lead is 57 times greater than that of cork.

We have learnt (§ 25) that the *weight of a body is proportional to its mass* (the amount of matter it contains); it is clear then that the density of a substance depends on the amount of matter

contained in a definite volume. It would therefore be more accurate to substitute the word *mass* for *weight* in the above paragraph.

The **Relative Densities** of substances are obtained by **comparing the masses of equal volumes** and the **density** of a substance must be expressed in **mass per unit volume.** Thus

the **density** of **lead** is 0·32 **ton** per **cubic foot,**
0·42 **pound** per **cubic inch,**
11·34 **grams** per **cubic centimetre.**

In what *units* shall we most conveniently express *mass* and *volume* and to what *substance* shall we assign *unit density*?

The **mass of 1 cubic centimetre of water** (4° **C.**) **is one gram:** it is therefore evident that if by weighing we find the mass in grams of 1 cubic centimetre of any substance we shall have a measure of its density relative to water (density = 1).

	Volume	Substance		Mass		Density in grams per c.c.
The mass of	1 c.c.	of water	is	1	gram	1
,, ,,	1 c.c.	of lead	,,	11·3	grams	11·3
,, ,,	1 c.c.	of gold	,,	19·3	grams	19·3
,, ,,	1 c.c.	of aluminium	,,	2·6	grams	2·6
,, ,,	1 c.c.	of copper	,,	8·9	grams	8·9
,, ,,	1 c.c.	of iron	,,	7·7	grams	7·7
,, ,,	1 c.c.	of brass	,,	8·3	grams	8·3
,, ,,	1 c.c.	of mercury	,,	13·6	grams	13·6
,, ,,	1 c.c.	of cork	,,	0·2	gram	0·2
,, ,,	1 c.c.	of alcohol	,,	0·8	gram	0·8
,, ,,	1 c.c.	of ice	,,	0·92	gram	0·92
,, ,,	1 c.c.	of sea water	,,	1·03	gram	1·03
,, ,,	1 c.c.	of petrol	,,	0·85	gram	0·85

The **Specific Gravity** of a substance is the mass of a volume of the substance divided by the mass of the same volume of water at 4° C.

In the above table it is clear that in expressing the *density in grams per c.c.*, we have also expressed the *specific gravity*, where water is chosen as the standard substance for a comparison of densities.

Connection between mass, volume and density. The *density* of a substance being *mass per unit volume*, if

M = *mass* of a body in *grams*, and

V = its *volume* in *cubic centimetres*, and

D = its *density* in grams per cubic centimetre, then

the mass of 1 c.c. $= D$ grams,

the mass of 2 c.c. $= 2D$ grams,

the mass of 3 c.c. $= 3D$ grams,

and the mass of V c.c. $= VD$ grams $= M$,

$$\therefore\ D = \frac{M}{V},$$

or in words, the **Density** of a body $= \dfrac{\text{its mass}}{\text{its volume}}$.

The **Relative Density** of a substance at any temperature is found by dividing its weight by the weight of an equal volume of water at the same temperature.

32. *Practical Exercises. Density by Measurement and Weighing.

1. You are given small cubes of brass, copper, iron, lead, aluminium, pine, deal and ebony. Measure the edge of each in cms.; calculate the *volume in c.c.s* and find (by weighing) the *mass in grams*. Arrange your results in columns and calculate the *mass of* 1 c.c. of each substance, *i.e.* its *density*.

2. Repeat, using spheres or cylinders of the same substances.

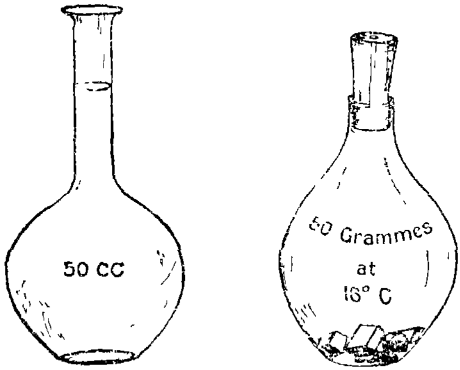
Example:

Substance	Solid	Measurements	Vol.	Mass	$\frac{\text{Mass}}{\text{Volume}}$ = Density
aluminium	cube	edge = 2 cm.	8 c.c.	21·58 gm.	$\frac{21 \cdot 58}{8} = 2 \cdot 69$ gm. per c.c.

3. Find the **density of an irregular solid** (*e.g.* a pebble or glass stopper). Obtain the *mass* in grams by weighing and the **volume** in c.c. by **displacement** [§ 23].

33. Use of Relative Density Bottle.

*Ex. i. **To find the Density of a Salt Solution.** In Fig. 38 two relative density bottles are shown which will hold a *definite volume* when filled—either (i) to a mark on the stem, or (ii) completely by means of a perforated stopper.



(i) Fig 38. (ii)

Since 1 gram of water occupies 1 c.c., the weight (in grams) of water which will fill the bottle will give its volume in c.c.

Take the following observations:

(1) Weight of bottle empty and dry 24·32 grams.

(2) ,, ,, full of water[1] 74·28 grams.

∴ Weight of water alone = **49·96** grams.

(3) Weight of bottle full of salt solution 83 27 grams.

∴ Weight of equal volume of salt solution = **58·95** grams.

The water weighs 49 96 grams and occupies 49 96 c.c.,

the salt solution ,, 58·95 grams ,, ,, 49·96 c.c.

∴ the density of the salt solution $= \frac{58\ 95 \text{ grams}}{49\ 96 \text{ c.c.}} = 1{\cdot}18$ grams per c.c.

[1] The bottle must be dry outside and be filled to the mark on the stem, if a perforated stopper is not provided.

***Ex. ii. To find the Density of Methylated Spirit.** Using the same bottle repeat observation (3), and find the weight of an equal volume of methylated spirit. On working out the result, the density will be found to be less than 1; methylated spirit therefore, if carefully poured, may be made to float on water.

***Ex. iii. To find the Density of Small Solids (e.g. shot).** First weigh the bottle filled with water; then, placing the shot in the scale-pan, *outside* the bottle, weigh the shot and full bottle together.

Record your results as follows:

(1) Weight of bottle full of water 74·28 grams.

(2) Weight of bottle + water + shot (*outside*) 97·69 grams.

∴ Weight of the shot = 23·41 grams.

Next place the full bottle on a clean saucer and carefully drop the shot into the bottle. The water which overflows has the same volume as the shot. Fill the bottle as before, dry the outside, and weigh again with the shot *inside*. The loss of weight observed is due to the displacement of water by shot. The volume of the shot is given in c.c. by noting the weight (in grams) of water displaced.

(3) Weight of bottle + water + shot (inside) 95·63 grams.

∴ Weight of water displaced by shot = (2) − (3) = 2·06 grams.

∴ Volume of the shot = 2·06 c.c.

$$\therefore \text{ Density of the shot} = \frac{23\cdot41 \text{ grams}}{2\cdot06 \text{ c.c.}} = 11\cdot34 \text{ grams per c.c.}$$

84. Alternative method for fragmentary solids, *e.g.* sand and powders.

Take the following observations:

(1) Weight of bottle empty and dry ....grams.

(2) „ „ + sand (inside) ....grams.

(3) „ „ „ + remainder of bottle space filled with water ....grams.

Empty out the contents, washing the bottle entirely free from sand, fill the bottle with water, dry the outside and find—

(4) Weight of bottle full of water ....grams.

$$\text{Then}\quad \text{Density of sand} = \frac{\text{Wt. of sand (grams)}}{\text{Vol. of sand (c.c.)}}$$

$$= \frac{\text{Wt. of sand (grams)}}{\text{Wt. of water displaced by sand (grams)}}$$

$$= \frac{(2) - (1)}{4 + (2 - 1) - 3}.$$

Examples V a (Density of Solids).

1. Find the density of (*a*) 50 c.c. of Tin weighing 365 gm., (*b*) 12 c.c. of Brass, weight 100·8 gm., (*c*) 20 c.c. of Lead, weight 228 gm., (*d*) 11 c.c. of Glass, weight 27·5 gm.

2. What will be the volume of (*a*) 5 gm. of Sulphur, density 2 gm. per c.c., (*b*) 2·5 gm. of Cork, density ·25 gm. per c.c., (*c*) 81 gm. of Marble, density 2·7 gm. per c.c., (*d*) 21 gm. of Silver, density 10·5 gm. per c.c.?

3. What will the following weigh: (*a*) 9 c.c. of Ivory, density 2 gm. per c.c. (*b*) 5 c.c. of Lead, density 11·4 gm. per c.c., (*c*) 4 c.c. of Glass, density 2·5 gm. per c.c., (*d*) a cube of Aluminium of 2 cm. side density 2·6 gm. per c.c.?

4. A rectangular block of pine wood measuring 20 cm. by 14 cm. by 15 mm. weighs 210 gm. Calculate the density of the wood.

5. A gold chain weighing 77·2 gm. is lowered into a graduated cylinder containing 38 c.c. of water and causes the level to rise to 42 c.c. Calculate the density of gold.

6. If the density of Ice is 0·918 gm. per c.c., what is the volume of 11 gm.?

7. What will a circular disc of zinc weigh if radius = 5 cm., thickness = 0·7 mm. and density = 7·2 gm. per c.c.?

8. 21 cm. of copper wire (density 8·9 gm. per c.c.) weigh 46·725 gm. Find the area of cross-section of the wire.

9. Find the diameter of a piece of brass wire if 1 metre of it weighs 1·056 gm. (density = 8·4 gm. per c.c.).

10. An ebony cylinder of diameter 1·6 cm., height 7 cm., weighs 15·696 gm. Calculate the density of ebony.

11. A cylinder of Lead (density 11·4 gm. per c.c.), diameter 1·4 cm., weighs 87·78 gm. Calculate its height.

12. A sphere of brass (density 8·4 gm. per c.c.) weighs 35·2 gm. What is its radius?

Examples V b (Relative Density of Liquids).

1. Calculate the density of (*a*) a decilitre of sea-water, weight 102·6 gm., (*b*) 20 c.c. of alcohol, weight 16 gm., (*c*) 30 c.c. of milk, weight 30·9 gm., (*d*) a litre of benzene, weight 880 gm.

2. What will the following weigh: (*a*) 100 c.c. of sulphuric acid, density 1·84 gm. per c.c., (*b*) 50 c.c. of olive oil, density 0·91 gm. per c.c., (*c*) 20 c.c. of ether, density 0·73 gm. per c.c., (*d*) a centilitre of mercury, density 13·6 gm. per c.c.?

3. What will be the volume of (*a*) 16 gm. of alcohol, density 0·8 gm. per c.c., (*b*) 270 gm. of oil, density 0·9 gm. per c.c., (*c*) 90 gm. of chloroform, density 1·5 gm. per c.c., (*d*) 51 gm. of aniline, density 1·02 gm. per c.c.?

4. An empty beaker weighs 44 gm. 20 c.c. of turpentine are run in and the beaker and contents weigh 61·6 gm. What is the density of turpentine?

5. A relative density bottle weighs 21·635 gm. when empty, 71·285 gm. when full of water, 62·645 gm. when full of methylated spirit. Calculate the relative density of the spirit.

6. A 50 c.c. flask weighs 70 gm. when full of water. What will it weigh when filled with alcohol of density 0·8 gm. per c.c.?

7. A decilitre flask full of sulphuric acid, density 1·84 gm. per c.c., weighs 220 gm. What would the flask weigh when empty?

8. If 40 c.c. of sea-water, density 1·025 gm. per c.c., are mixed with 60 c.c. of water, what is the density of the mixture?

9. How many c.c. of water must be mixed with 100 c.c. of sulphuric acid (density 1·84 gm. per c.c.) that the density of the mixture may be 1·2 gm. per c.c.?

10. 12·6 gm. of glycerine (density 1·26 gm. per c.c.) are mixed with water and the density of the mixture is 1·1 gm. per c.c. How much water is present in the mixture?

11. Find the density of a mixture of 64 gm. of alcohol, density 0·8 gm. per c.c., and 74·9 gm. of acetic acid, density 1·07 gm. per c.c.

12. 22 gm. of salt (density 2·2 gm. per c.c.) are dissolved in 90 c.c. of water. Calculate the density of the solution.

13. A bottle weighs 115 gm. when full of water, 99 gm. when filled with alcohol of density 0·8 gm. per c.c. What will the empty bottle weigh?

EXAMPLES V C (RELATIVE DENSITY OF FRAGMENTARY SOLIDS).

1. A relative density bottle filled with water weighs 49 gm. 7·2 gm. of iron nails are dropped in, water being allowed to overflow. The bottle and contents are now found to weigh 55·2 gm. Calculate the relative density of the nails.

2. A relative density bottle weighs 40 gm. when full of water. 5 gm. of glass beads are poured into the bottle, water being allowed to overflow. The bottle and contents now weigh 43 gm. Find the relative density of glass.

3. A 50 c.c. flask weighs 20 gm. when empty. 10 c.c. of aluminium powder (relative density 2·6) are poured into it and it is filled up with water. What will it now weigh?

4. Find the relative density of sand from the following: Weight of flask, 20 gm.; weight of flask and sand, 25 gm.; flask and water, with sand inside, 72 gm.; flask full of water only, 70 gm.

5. Find the relative density of shot from the following: Weight of flask, 20 gm.; flask and shot, 43·94 gm.; flask, with shot inside, filled with water, 91·84 gm.; flask full of water only, 70 gm.

SECTION II.

HYDROSTATICS

CHAPTER VI.

PROPERTIES OF MATTER.

35. Force.

In considering the properties of matter we shall constantly use the word—*force*. What is a *force*? The inability of matter to change its state of rest or of motion (§ 24) is called its **inertia**—*forces* are necessary to change that state. Let us give an instance. A stone resting on level ice requires *force* to put it in motion, but once moving, if the ice is perfectly smooth and if there is no air to cause resistance, the stone will continue to move at a uniform speed, unless some *force* either quickens its pace or lessens it until finally the stone is brought to rest. In reality, the air presses against the stone and friction drags on it: both *forces* of *pressure* and of *friction* help to stop it.

General properties of matter. In § 24, it was stated that, in addition to possessing *inertia*, *matter occupies space* and *attracts* other forms of matter (*i.e.* it *has weight*). In our study of Chemistry we find that *matter is indestructible* (*Exp. Chem.* § 34). **Matter exists in three states**—*solid*, *liquid* and *gaseous*. **Solids** possess **rigidity**, *i.e.* they offer resistance to forces which tend to change their shape or their volume. **Liquids** and **Gases**, which are classed together as **Fluids**, offer no lasting resistance to such forces.

In the study of **Hydrostatics** we investigate how forces act on *fluids* and keep them at rest.

36. The Constitution of Matter.

The gaseous state. We know that ice (a solid) may be turned into water (a liquid) by heat and that further heating converts water into steam (a gas). The reverse process of cooling changes steam to water and water to ice. The effect of heat on all solids is to convert them finally into the gaseous state. The effect of cooling gases is finally to convert them to the solid state. This solar system of ours consisting of the sun, the planets with their moons and the asteroids is supposed to have been once gaseous. It is losing heat and finally will all become solid; for instance, the present oceans of our earth will be ice throughout and on them at first will rest an ocean of liquid "air" which further cooling will convert to the solid state. It is simplest therefore to begin our study of the constitution of matter by considering the gaseous state, where matter "is without form."

Matter is supposed to be made up of little particles called **molecules** (Lat. = little masses), which are far too small to be seen by the most powerful microscope. These molecules are said to be in constant motion and, in the gaseous state, have free paths in space. They strike against each other and against the walls of the vessel which contains them, thus producing a constant battering or bombardment which causes pressure (*e.g.* in an inflated football or toy balloon). The molecules are supposed to be perfectly **elastic,** *i.e.* they lose none of their energy of rebound when they collide with any object; consequently the pressure caused by their bombardment remains constant if the conditions are unchanged. Gases are easily compressed and the pressure and density rise as the molecules are more confined. Gases possess no rigidity, as stated above, consequently they can have no definite shape, and they completely *fill the vessel* in which they are placed. Gases readily mix with or **diffuse** into each other.

Exp. Pull out the piston of a bicycle pump, close the nozzle with your finger and push in the piston. The air within is easily *compressed*, but on

releasing the piston, it flies back to its original position, thus showing the *elasticity* of the air as well as the *pressure* of the confined gas.

37. The liquid state.

A gas by compression and cooling may be converted into a *liquid*; its volume is much reduced in the process, thus showing that the molecules are more tightly packed into a given space in a liquid than in a gas. In the reverse process, as for instance when water is boiled, the steam occupies many hundred times the volume of the water from which it is obtained. Liquids however are practically *incompressible* and do not expand except under the action of heat, consequently they have a definite volume, but as they possess no lasting rigidity they have *no definite shape*. This absence of rigidity causes them to flow in a vessel and fit themselves to its shape but the upper or *free surface is horizontal*. A liquid is said *to find its own level*, thus, if a number of vessels (Fig. 39) communicate with each other and a liquid is poured in, the level LL_1 of the free surfaces will be horizontal and the same for all. This property is used in levelling

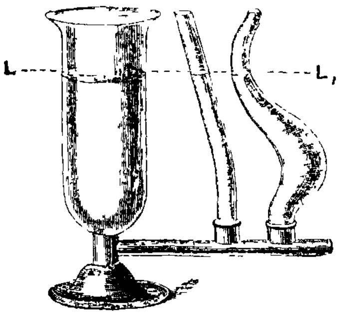


Fig. 39.

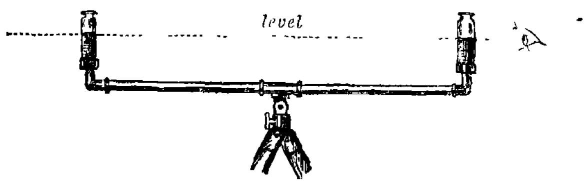


Fig. 40.

instruments (Fig. 40) and in supplying towns with water from a reservoir situated at a higher altitude than the houses of the town.

Some liquids *pour* more readily than others; they are said to differ in **viscosity.** Thus treacle, lava, honey and seccotine are more *viscous* than water and petrol. Pitch (cobbler's wax), vaseline, train-oil and soft-soap are highly viscous liquids.

38. Cohesion, Surface Tension, Capillarity, Diffusion.

The force of attraction between the molecules of a body is called **cohesion.** Liquids exhibit slight cohesion, gases none. This attraction of the molecules for each other and the absence of rigidity cause a small drop of a liquid to assume a *spherical* shape.

At c the centre of the drop (Fig. 41), a molecule is pulled or attracted in all directions by other molecules surrounding it; but at or near the surface (A, A') a molecule will be pulled by cohesion towards the interior of the drop, the forces being evenly distributed about the radius $A'c$ at the point A'; hence there is a constant tendency for the drop to contract radially towards the centre. At any point on the surface (A) there will be forces equal but opposite in direction (T, T), so that the surface behaves as though it were covered by a thin skin pulled tightly in every direction. This pull along the surface, called **surface tension,** is clearly seen in a soap bubble. The bubble slowly collapses if the enclosed air has access of escape. Surface tension pulls the drop into the shape of the solid which has least surface for a given volume, viz. that of a *sphere.* If the attraction of the molecules for each other (*cohesion*) is less than their attraction to the sides of the vessel containing the liquid (*adhesion*), the liquid wets the vessel, as for instance is the case with water in a glass: with mercury however in a glass vessel the reverse is seen—mercury

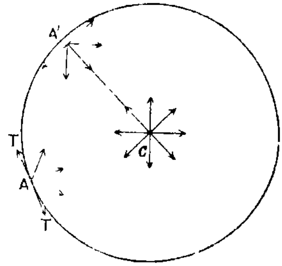
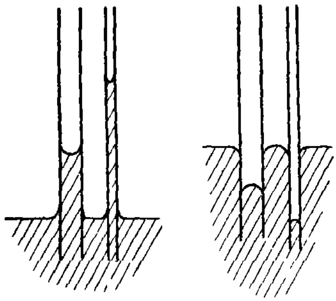


Fig. 41.

does not wet glass because attraction of cohesion in mercury is greater than that of adhesion between mercury and glass.

Exp. Capillarity. Heat pieces of glass tubing in the blowpipe flame and draw them out into fine capillary tubes (*Exp. Chem.* § 3). Hold the ends of these tubes in (1) coloured water, (2) coloured alcohol, (3) mercury. Note that the liquid rises in the tubes in water and alcohol but is depressed in the case of mercury. These results are said to be due to *capillarity* (Lat. *capilla*: a hair) where the pull upwards or downwards is due to surface



Capillary tubes in water or alcohol.

Capillary tubes in mercury.

Fig. 42.

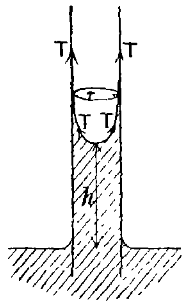


Fig. 43.

tension (Fig. 42). The finer the tube the higher the liquid rises[1]. In the wick of a lamp the oil rises because of capillary attraction.

[1] Let T be the surface tension in grams across 1 cm. length. In a capillary tube of radius r (Fig. 43) placed in water which rises to a height h, there is a pull of capillarity up the tube of

$$T \times \text{circumference of tube} = 2\pi r T \text{ grams},$$

which supports the column of water of

$$\text{volume} = \pi r^2 \times h \text{ c.c.}$$

and of

$$\text{weight} = \pi r^2 h \text{ grams},$$

$$\therefore\ 2\pi r T = \pi r^2 h,$$

$$\therefore\ \frac{2T}{r} = h,$$

$\therefore$ the smaller r becomes the greater h becomes.

Exp. Porosity and Diffusion. Having closed one end of each of two pieces of ⅜ inch tubing with thin plugs of Plaster of Paris paste, allow the paste to set and dry for some hours. To all appearances the plugs are solid; they may however be shown to be *porous* (*i.e.* traversed by minute passages between the molecules) as follows: (1) Fill one of the tubes with coal-gas and place it vertically with its open end dipping below the surface of water in a beaker (Fig. 44). The coal-gas escapes through the plug more rapidly than air can enter; water consequently rises in the tube. After some hours air will have completely replaced the coal-gas. (2) The reverse process is shown by supporting the second tube vertically with its end dipping under the water, and surrounding it by a cylinder full of coal-gas. Gases pass in both directions (*diffuse*) through the plug but coal-gas enters more rapidly than air escapes, hence bubbles come out at the bottom of the tube.

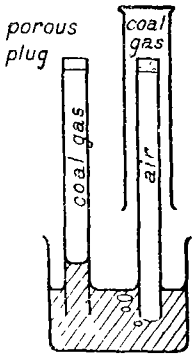


Fig. 44.

39. The Solid State.

Solids possess **rigidity** (§ 30) to a varying extent; steel, for instance, is more rigid than copper, and copper than india-rubber. Solids, like liquids, are only very slightly *compressible.* Solids vary in their capacity for recovery of shape after being squeezed, twisted or pulled: this property of recovery is termed the **elasticity** of the substance. Gold, silver and copper may be drawn out in fine wire and are said to be **ductile**; such metals are as a rule also **malleable,** *i.e.* they may be hammered into thin sheets; gold leaf has been estimated to have a thickness of less than $\frac{1}{250,000}$ inch.

Exp. Tie two long thin wires of copper and of steel respectively, of equal length and thickness, to a beam at the top of a high room. Suspend a scale-pan from each wire and place weights in the pans. Note that the distance of the pans from the floor lessens as weights are added. Remove the weights; there is recovery in both wires, as regards length, provided only small weights have been added. On the addition of more weights the copper wire becomes permanently stretched and is said to have passed its

limit of elasticity, but the steel wire recovers. Further addition of weights breaks the steel wire but the copper wire will probably be drawn out into a still finer wire until the pan reaches the floor, thus showing that copper, although less *elastic* than steel, possesses *ductility* to a much greater extent.

Evaporation of liquids and the pressure which their vapours exert is referred to in the section on Heat and also in *Exp. Chem.* §§ 17, 29. Diffusion of gases has already been mentioned in this chapter. In liquids, the molecules mix less rapidly even when the liquids are stirred; in *solids, diffusion* has been proved to take place but to a very limited extent and only after long lapse of time. It has been proved that, if a piece of pure lead and a piece of pure gold are kept in close contact and long afterwards separated, the lead contains traces of gold and the gold contains traces of lead. It is also a well-known fact that ice lessens in volume by evaporation, although it is kept solid at a temperature below its melting point, thus showing that even in solids, the molecules are in motion and that near the surface they may leave the solid altogether and either pass away into the atmosphere or penetrate the interstices or spaces between the molecules of an adjacent solid.

EXAMPLES VI.

1. Explain the difference between Solids, Liquids and Gases.

2. What practical use is made of the fact that 'water finds its own level'?

3. Why is the surface of the mercury in a barometer curved?

4. Prove that capillarity will cause a liquid to rise higher in a narrow tube than in a wide one.

CHAPTER VII.

FLUID PRESSURE. PRESSURE IN LIQUIDS.

40. How to measure pressure.

The **pressure** exerted by **the wind** may be measured by the following method:

Ex. i. Suspend a square kite (K) by one of its corners in a steady breeze and tie the four bands from the laths of the kite to a string in such a way that the plane of the kite is at right angles to the main string which is exactly in the direction of the wind (Fig. 45). Attach a spring balance S (dynamometer) to the string and, steadying the kite, read the pull (F) registered by the dynamometer. Measure the area (A) of the kite. Then the *whole pressure* or *thrust* of the air is shown by the force (F) distributed over the whole area (A) of the kite.

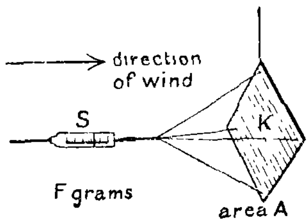


Fig. 45.

Then the **pressure on unit area** $= \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$.

If the pull on the dynamometer $= 16$ pounds,
and the area of the kite $= 4$ sq. ft. $= 576$ square inches,
then the pressure $= \frac{16 \text{ pounds}}{576 \text{ sq. ins.}} = \frac{1}{36}$ *pound per square inch.*

The **pressure** due to a **column of liquid** may be measured as follows:

Ex. ii. A gas cylinder (C), the closed end of which has been cut off, is clamped vertically. A ground glass plate (P) is held firmly pressed against

the lower ground rim of the cylinder by means of a string attached to a spring balance (S) which is pulled tightly (Fig. 46). Pour *water* into the cylinder and note the height of the column (h cm.). Then gradually lessen the force supporting the plate by lowering the dynamometer. Note the pressure when the support gives way and the water escapes. Repeat several times, always filling the cylinder with water to the same level. Subtract the weight of the plate from the average of your results. Let F = the supporting force. Measure the internal diameter of the cylinder and calculate the *area of cross-section* (A sq. cm.). Then the pressure on a plate of A sq. cms. is F grams, *i.e.* the **pressure** $= \frac{F}{A}$ **grams per square centimetre.** This result $\left(\frac{F \text{ grams}}{A \text{ sq. cms.}}\right)$ clearly gives us the pressure when the area of cross-section of the cylinder is reduced to 1 *sq. cm.* and, since the height of the cylinder of water $= h$ cm., the number of c.c. of water pressing on each sq. cm. $= h$ c.c., but 1 c.c. of water weighs 1 gram.,

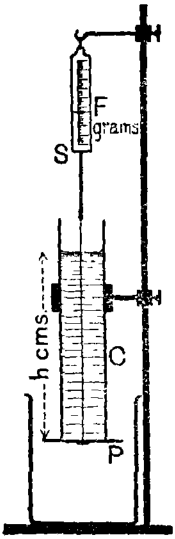


Fig. 46.

$\therefore$ the pressure due to a column of water of h cm vertical height $= h$ grams per square centimetre, and we find that in a liquid the **pressure is proportional to the depth.**

Example Observations.

Weight of plate	$= 18$ grams.
Average pull on dynamometer	$= 750$ grams.
Force necessary to support water (F)	$= 750 - 18 = 732$ grams.
Depth of water (h)	$= 30$ cm.
Internal diameter of cylinder	$= 5{\cdot}6$ cm.

$\therefore$ Area of cross-section $= \pi r^2 = 24{\cdot}64$ sq. cm.

$$\therefore \text{ Pressure} = \frac{732 \text{ grams}}{24{\cdot}64 \text{ sq. cm.}} = 29{\cdot}7 \text{ grams per sq. cm.}$$ [1]

[1] **Percentage Error.** If we assume that in water the pressure in grams per square centimetre at a depth h cm. $= h$ grams, then the pressure at depth 30 cm. $= 30$ grams per sq. cm.

By experiment we have found the pressure $= 29{\cdot}7$ grams per sq. cm.

$\therefore$ the experimental error $= 30 - 29{\cdot}7 = {\cdot}3$ in the correct result which is 30

$$\therefore \text{ the } \textit{percentage error} = \frac{{\cdot}3 \times 100}{30} = 1\ \%.$$

EXP. A graphic method of showing that in a liquid the pressure increases as the depth increases is shown in Fig. 47, where a tall vessel is

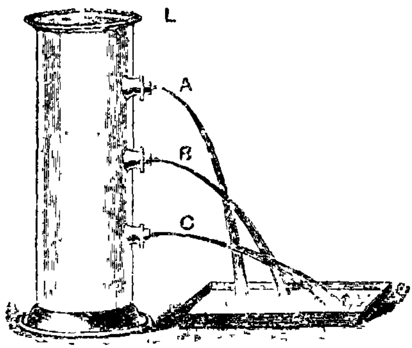


Fig. 47.

furnished with three outlets at different levels. If the outlets are of the same bore (cross section), more water will flow from a lower than from an upper level in a given time. A tap above the vessel should give a sufficient supply of water to keep the *head of water* at level L constant. Collect and measure the volume of water which flows from each tap in 15 secs.

Pressure at depth h in a liquid of density D.

Since, by definition, 1 c.c. of the liquid weighs D grams, the weight of a column of liquid of h cm. vertical height and 1 square cm. cross-section $= h \times D$ grams.

$\therefore$ pressure at depth $h = h \times D$ **grams per square cm.**

41. Fluids (Liquids and Gases) transmit pressure equally in all directions.

(1) If we place our hands some distance apart on a partially inflated bicycle tyre and press gently with the fingers of one hand we can feel the pressure transmitted through the air in the tube to the other hand.

(2) If we squeeze an india-rubber ball which we have filled with water through a single hole by suction, we know that the same amount of pressure applied at any part of the ball towards

its centre produces the same effect in squirting water out of the hole, no matter in what direction the hole is pointing. We can imagine the ball fitted with outlet tubes of equal cross-section (1 sq. cm.) in which move frictionless pistons *P*, *Q*, *R* (Fig. 48). If an inward pressure of *F* grams is applied at *P* there will be transmitted to all parts of the cover a pressure of *F* grams per square centimetre and consequently the pistons *Q* and *R* will each be driven out with a force of *F* grams. On this principle hydraulic presses and lifts are constructed (see § 66).

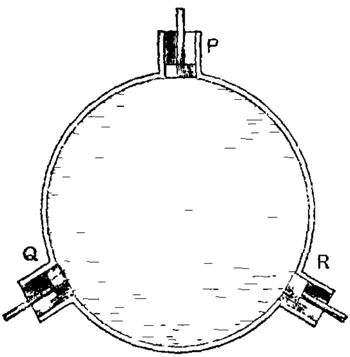


Fig. 48.

42. To show experimentally that in a liquid—

(*a*) **the pressure is the same at equal levels, and**

(*b*) **that the pressure is the same in all directions.**

Exp. In § 33, the statement was made that in a liquid the pressure is proportional to the depth and since we assume the surface to be level (*i.e.* in a plane at right angles to the direction of the plumb-line) it follows that the pressure will be the same at equal levels. Pour water into a large beaker capable of holding several litres and mark the level *LL*, with a chalk line or with bands of paper, when the depth is about 2 inches (Fig. 49). Fill up the beaker to a depth of about 10 inches. Over a funnel (*F*), bent in the form of an L, stretch and tie tightly a piece of thin india-rubber; mark the centre of the rubber circle with a spot of ink or dye. Connect the open end of the funnel to a U-tube, in the bend of which is placed some coloured water. Lower the rubber-closed funnel into the beaker and note that the pressure on the rubber is *transmitted* through the air in

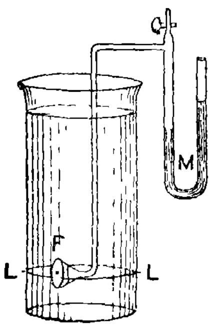


Fig. 49.

the U-tube and that the coloured water is depressed on one side and rises equally on the other, and that the deeper the funnel is lowered the greater is the amount of pressure recorded. This U-tube is a simple form of *pressure gauge* or *manometer M*.

Move the funnel *pointing it in various directions so that its centre is at different positions* in the level *LL* marked by the chalk or paper line. It will be seen that if these conditions are carefully maintained, the pressure as shown by the manometer does not vary.

The pressure at a point is measured by the thrust on a square centimetre having the point at its centre.

The shape of the vessel does not affect the pressure at any particular depth.

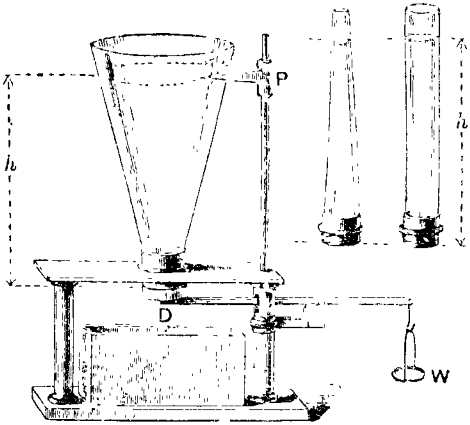


Fig. 50.

Exp. The three variously shaped vessels shown in Fig. 50 have openings at the bottom of *equal area of cross-section*. The edge of the opening is carefully ground and a plate attached to one arm of a lever is pressed against the opening by adding weights *W* to the scale-pan. Water is run into the vessel until the supporting plate yields and the water escapes. The depth of water (*h*) is noted and the experiment is repeated when another of the vessels is placed in position against the plate. Water to the same depth (*h*) is added before the plate moves from the rim and the scale-pan is raised, thus showing that the pressure on a definite area of the base depends on the depth of the liquid and not on the shape of the vessel.

43. Revision Experiment. (A.)

To show that the upward pressure (or thrust) at any depth in a liquid is equal to the downward pressure.

(1) Procure two or three lamp chimneys of entirely differing shapes. (The lower edge should be ground flat.) In Fig. 51 *a* a cylindrical one (*C*) is shown. *P* is a piece of mill-board[1], which, if it can be obtained, should be of the same density as water. The mill-board is wetted and held tightly against *C* by a string passing through its centre. Lower the apparatus into water. The mill-board remains in position *pressed upwards* when the string is loosened. Carefully pour water into chimney and note that the mill-board comes away when the water reaches *L*, the level of the surrounding liquid.

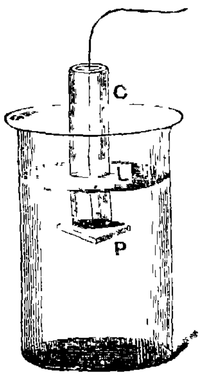


Fig. 51 *a*.

(2) Repeat the experiment, substituting a chimney of different shape, and lowering to various depths.

(3) Repeat the experiment, holding the chimneys in positions other than the vertical.

Draw your conclusions and write them out in order.

Revision Experiment. (B.)

To find **the relation between** (*a*) **Pressure,** (*b*) **Depth and** (*c*) **Area** over which pressure is exerted in a liquid[2].

[1] A flat disc of box-wood, of the same density as water, serves the purpose.

[2] In this Exp. it is assumed that when a cylinder *floats* vertically in a liquid, the pressure exerted by the liquid upwards on the base (upthrust) equals the weight of the floating body. For exercises on this Exp. see Examples on Chap. VII.

*Exp. i. Flatten the bottom of a long test tube by softening the rounded part in a blowpipe flame and pressing on an iron plate. Put a slip of mm. squared paper, marked off in centimetres measured from the outside of the bottom of the tube, into the tube, which must be corked (Fig. 51 *b*). Load the tube with sand or shot until it will float vertically in a jar of water. Note the position of the water-level by means of the squared paper scale and measure h_1. Remove the tube, dry it and weigh it with its contents (w_1). Add more sand or shot to sink the tube to a lower level, note h_2, and weigh again (w_2).

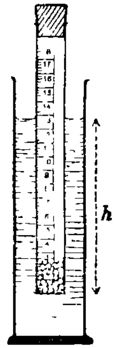


Fig. 51 *b*.

Find the ratio $\frac{w_1}{h_1}$ and $\frac{w_2}{h_2}$,

i.e. the relation between weights (pressure of water at different depths) and the depths.

*Exp. ii. Prepare and load a boiling tube similarly, until it floats with *the same lengths* immersed as in Exp. i. Dry and weigh as before. Measure the diameters of the two tubes, calculate the area of cross-section of each, and find the relation between weights and areas, *i.e.* the ratio $\frac{W}{A}$.

State your deductions from each experiment.

44. Pressure in Liquids at rest. (*Revision and memoranda.*)

1. Density being mass per unit volume and weight being proportional to mass, density is a measure of weight per unit volume.

2. The free surface of a liquid is level (*i.e.* at right angles to the plumb line).

3. The pressure at a point is measured by the thrust on a sq. cm. having the point as centre.

4. The pressure is proportional to depth beneath the surface and is therefore the same at the same horizontal levels. In a liquid of density D grams per c.c. and at depth h, the pressure $= h \times D$ grams per sq. cm.

5. The pressure at a point is the same in all directions, consequently at any point to every pressure there must be an equal but opposite pressure, for otherwise the liquid would move.

6. Fluids transmit pressure equally in all directions.

45. To find the Relative Density of Liquids by "Balancing columns"—"U-tube method."

***Exp. i. To find the density of a salt solution.** Balance a column of the solution with a column of water as follows. Bend 2 feet of $\frac{3}{8}''$ glass tubing at its middle point in the form of a U (*Exp. Chem.* § 8). Fill the bend with mercury to about 1" in depth and clamp the tube vertically. Pour salt solution into the right-hand arm S (Fig. 52) and water into the left-hand arm, *a little at a time*, until the arms are nearly full and the *mercury is at the same level* $L_1 L_2$ on both sides. Measure[1] the vertical heights of the two columns. For instance, suppose that—

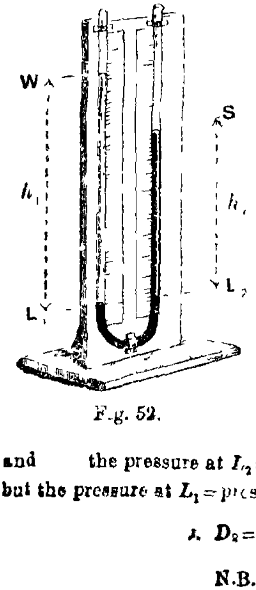


Fig. 52.

Height of water column h_1 $= 18·1$ cm.,
Height of salt solution column $h_2 = 15·2$ cm.

Let D_1 = density of water
and D_2 = density of salt solution.

Then

the pressure at $L_1 = h_1 \times D_1$
$= 18·1 \times 1$ grams per sq. cm.,

and the pressure at $L_2 = h_2 \times D_2 = 15·2 \times D_2$ grams per sq. cm.,
but the pressure at L_1 = pressure at L_2 ($L_1 L_2$ being level).

$$\therefore D_2 = \frac{18·1}{15·2} = 1·19 \text{ grams per c.c.}$$

N.B. $h_1 \times D_1 = h_2 \times D_2$,

$$\therefore \frac{D_1}{D_2} = \frac{h_2}{h_1},$$

or, in words,—*the densities* of the two liquids *are inversely proportional to the heights of the columns* which balance each other.

***Exp. ii. Find the relative density of two liquids which do not mix.** Use the same method and apparatus to find the density of turpentine relative to that of water. In this case mercury may be dispensed with. Half fill the U-tube with water and then carefully pour turpentine into one side. *Measure* the vertical heights of the columns *from the level where the two liquids meet.* Below this level of separation the water, which is the denser liquid, acts the part of a balance just as the mercury did in Exp. i.

[1] See *Exp. Chem.*, Fig. 9, § 8, on "Parallax Error."

EXAMPLES VII.

1. A test-tube loaded with shot weighs 20 gm. When floated in water it sinks to a depth of 10 cm. If two more grams of shot are added, to what depth will it sink?

2. A loaded test-tube weighing 20·44 gm. floats in water with 11·2 cm. below the surface. What extra weight will be needed to sink it 12 cm.?

3. A test-tube of diameter 1·6 cm. and a boiling tube of diameter 2·2 cm. are loaded with sand till they float vertically with the same length immersed in water. If the weight of the test tube is 20·8 gm., what does the boiling-tube weigh?

4. A test-tube and boiling-tube weighing 16 gm. and 36 gm. respectively sink to the same depth in water. If the diameter of the boiling-tube is 2·4 cm., find diameter of test-tube.

5. What is the pressure per sq. cm. at the base of a column of mercury 76 cm. high (density of mercury = 13·6 gm. per c.c.)?

6. Calculate the "thrust" (total pressure) on the bottom of a tank full of water, 6 dm long, 40 cm. wide and 40 cm. deep. What will be the pressure *at any point* in the base?

7. If the town water reservoir is 600 ft above sea-level, find the pressure per sq in. in a pipe at the top of a building 24 ft. above sea-level.

8. Some mercury is poured into a U-tube; water is then poured into one limb till it measures 34 cm. How high will the mercury in the other limb stand above the common surface?

9. If the density of sea-water is 1·025 gm. per c.c., what is the pressure per sq. cm. at a depth of 680 metres in the sea? At what depth in mercury would the pressure be the same?

10. What is the height of a building if the pressure at taps on ground floor and top floor is 30 lbs. and 15 lbs. per sq. in. respectively?

11. Water is poured into a U-tube and olive oil added. The height of the oil above the table = 19 cm., that of the water = 17·65 cm.: the common surface 4 cm. Calculate the relative density of oil.

12. The thrust on the bottom of a beaker (diameter 7 cm.) when full of mercury is 5·236 kgm. What is the height of the beaker?

13. Methylated spirit (density 0·816 gm per c.c.) is poured into a U-tube containing mercury until the latter stands 3 cm. above the common surface. What is the length of the spirit column?

CHAPTER VIII.

PRINCIPLE OF ARCHIMEDES.

46. The Principle of Archimedes.

We are well acquainted with the fact that when we are bathing the water buoys us up. In learning to swim very little support is required under the chin or arms to keep our heads above water. We also know that the more we are immersed the less effort is required to support us, the water exerting an upward force which almost counteracts our weight. These facts were observed about 200 B.C. by a philosopher of Syracuse named Archimedes. The principle named after him is as follows—**When a body is wholly or partially immersed in a fluid, it loses weight by the weight of fluid displaced,** or in other words **there is an upthrust on the body equal to the weight of the fluid displaced.**

A little thought will convince us that this statement is true.

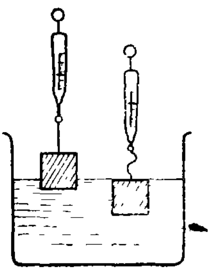


Fig. 53.

Imagine that we have a *magic* cubic centimetre box with walls so thin that they occupy no space and so light that they weigh nothing. Imagine that we fill this box with water and tie it by a weightless string to a spring balance. The balance will register the weight of 1 c.c. of water, viz. 1 gram. Into a bowl of water lower the box until one quarter of its volume is under the surface (Fig. 53): this one quarter, weighing $\frac{1}{4}$ gram, becomes

part of the water in the bowl except that it is separated from it by a weightless partition of no thickness. The balance will therefore register ¼ gram less, or in other words the boxed up water has *lost weight by the weight of water displaced.* Continue to lower the box until it is quite immersed. The balance will now register *zero*, for the whole gram of enclosed water has become part of the water in the bowl except that it is separated from it by the weightless box. Next let us fit into the imaginary box 1 c.c. of lead which weighs 11·3 grams. Lower the leaden cubic centimetre into water. The lead continues to lose weight until it is completely immersed, when again 1 c.c. of water (1 gram) is displaced and the spring-balance registers 10·3 grams.

47. Experimental Proof of the Principle of Archimedes.

Exp. The following is often called the "cylinder and bucket" experiment (Fig. 54). A *cylinder C, fitting exactly into a bucket B*, is suspended to one arm of a balance and counterpoised. The cylinder is then placed in a beaker, the latter being supported on a shelf placed over the scale-pan. Water is then carefully poured into a beaker until it is about full. The balance is disturbed as part of the weight of the cylinder is supported by the water. Water is now added to the bucket until the balance again swings evenly: it is then *found* that the same volume of water has been poured into the bucket as the cylinder is displacing in the beaker. Fill the beaker nearly full of water and continue to add water to the bucket until a balance is again obtained: the bucket will now be found to be exactly full, and, as its internal volume equals exactly the volume of the cylinder, it is evident that *on being placed in water the cylinder has lost in weight by the weight of the volume of water displaced.*

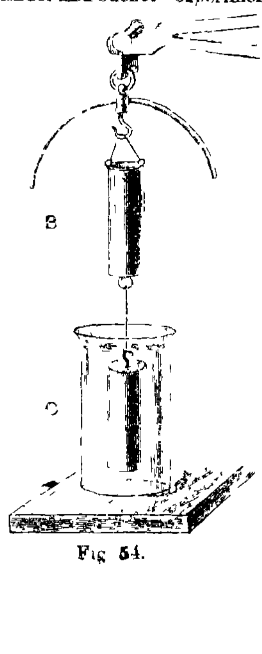


Fig 54.

48. To find the Relative Density of a Solid by the Principle of Archimedes.

***Exp. i. To find the Relative Density of Copper.** Suspend a copper cylinder *C* by a piece of fine silk to one arm of a balance (Fig. 55).

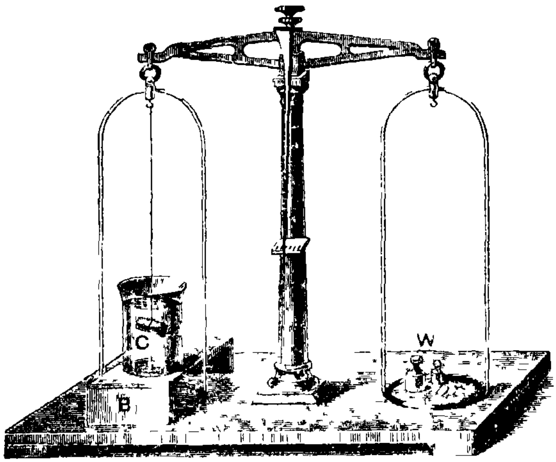


Fig. 55.

(1) Weigh the copper cylinder $= 64{\cdot}29$ grams.

Place the cylinder in a beaker of water, the beaker being supported on a bridge *B*.

(2) Weigh the copper cylinder (in water) $= 57{\cdot}11$ grams.

The loss of weight, on weighing in water, $= 7{\cdot}18$ grams.

∴ by the Principle of Archimedes,

the weight of water displaced $= 7{\cdot}18$ grams.

∴ the weight of water equal in vol. to the cylinder $= 7{\cdot}18$ grams.

$$\text{The Rel. Density of Copper} = \frac{\text{Wt of copper cylinder}}{\text{Wt of an equal volume of water}}$$

$$= \frac{64{\cdot}29 \text{ grams}}{7{\cdot}18 \text{ grams}} = 8{\cdot}95;$$

hence, the rule:

$$\textit{Relative Density of a Solid} = \frac{\textbf{Weight of solid (in air)}}{\textbf{Loss of weight in water}}.$$

***Exp.** ii.* Repeat the experiment, using a *spring-balance.*

Further Practical Exercises. Find the density of Iron (cylinder), Aluminium *(cylinder)*, and *Glass* (a *stopper*). *(Cf. § 31.)*

49. To find the Relative Density of a Liquid by the Principle of Archimedes.

***Exp.** To find the Relative Density of Methylated Spirit.*

$$\text{Rel. Density of Spirit} = \frac{\text{Wt. of a given volume of spirit}}{\text{Wt. of same volume of water}} \quad \text{(by Definition).}$$

Weigh a metal cylinder (1) in air, (2) in methylated spirit, (3) in water, then,

$$\text{Rel. Density of Spirit} = \frac{\text{Loss of wt. of a solid in spirit}}{\text{Loss of wt. of same solid in water}} \quad \text{(by Archimedes)}$$

$$\text{Rel. Density of Spirit} = \frac{\text{Wt. in air} - \text{wt. in spirit}}{\text{Wt. in air} - \text{wt. in water}}.$$

For example, take the following observations:

(1) Wt. of a metal cylinder in air $= 64{\cdot}29$ *grams.*

(2) Wt. of the *same* cylinder in spirit $= 58{\cdot}55$ grams.

(3) Wt. of the *same* cylinder in water $= 57{\cdot}11$ grams.

$$\therefore \text{Rel. Density of Spirit} = \frac{64{\cdot}29 - 58{\cdot}55}{64{\cdot}29 - 57{\cdot}11} = \frac{5{\cdot}74 \text{ grams}}{7{\cdot}18 \text{ grams}} = 0{\cdot}8$$

50. To find the Relative Density of a Solid which floats on water.

***Exp.** To find the Relative Density of Cork.* Weigh a metal cylinder, round which a piece of thin wire is wound, in water as in Exp. i, § 48, Fig. 55.

(1) Weight of cylinder + wire (in *water*) (Fig. 55) $= 67{\cdot}54$ grams.

Place the cork in the balance-pan (dry) and weigh again.

(2) Weight of cylinder + wire (in *water*) + cork (in *air*) $= 67{\cdot}99$ grams.

$\therefore$ Weight of cork $= (2) - (1)$ $= 0{\cdot}45$ grams.

By means of the wire, tie the cork to the cylinder, and weigh both cylinder and cork in water: the loss of weight = weight of water displaced by the cork = weight of water equal in volume to the cork.

(3) Weight of cylinder + wire + cork (*all in water*) = 65 74 grams.

$\therefore$ Weight of water displaced by the cork = (2) − (3) = 2·25 grams.

$\therefore$ Rel. Density of Cork $= \frac{\text{Wt of cork}}{\text{Wt. of equal vol. of water}} = \frac{0\ 45}{2\ 25} = 0\ 2.$

51. To show that when a solid is immersed there is a downthrust on the bottom of the vessel equal to the upthrust of the liquid on the solid[1].

(1) Weigh a beaker containing water = 82 67 grams

(2) Suspend the same copper cylinder as was used in § 48, Exp i, in the beaker (Fig 56) and weigh again = 89 85 grams

There is an *increase* of weight observed = 7 18 grams,

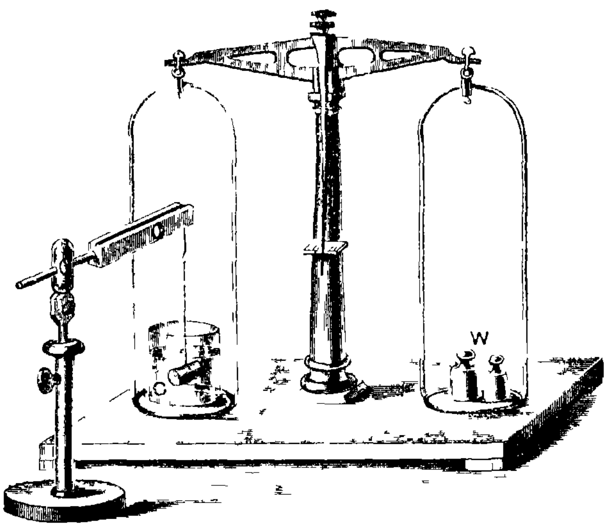


Fig 56.

[1] This experiment is a good illustration of Newton's Third Law—"To every action there is an equal and contrary reaction"

which also equals the *loss* of weight recorded on weighing the cylinder in water (§ 48, Exp. i). There is therefore a downthrust on the bottom of the vessel equal to the weight of water displaced, *i.e.* equal to the upthrust of the liquid on the solid immersed. This result was to be expected, for by suspending the solid in the water we have *raised the level of the water* in the beaker by the volume of the solid. The same result would have been obtained if, instead of suspending the solid, we had raised the surface of the water to the new level by addition of water equal in volume to that of the solid.

***Revision Experiment. Find the Relative Density of Methylated Spirit** by the method of the last experiment (downthrust method). Find the downthrust (1) in spirit, (2) in water, and explain fully how the relative densities of the two liquids may be obtained.

*Practical Exercises in indirect measurement.

1. Find the *diameter* of a piece of wire by finding the volume of a known length by a displacement method and hence obtain the *area* of cross section. Confirm your result by using the screw-gauge (§ 6).

2. Find the *length* of a coil of wire, without unwinding it.

3. Find the average *internal diameter of a glass tube*, given enough mercury (density 13·6) to fill the tube, a beaker, cm. scale, balance and weights.

Examples VIII (Archimedes' Principle).

1. A piece of copper weighs 40·05 gm. in air and 35·55 gm. in water. Find the relative density of copper.

2. If a piece of brass (density 8·4 gm. per c.c.) weighs 68·04 gm. in air, what will it weigh in water?

3. Find the density of a piece of marble, volume 6·55 c.c., which weighs 17·685 gm.

4. If a man can only just lift 144 kgm. of iron (density 7·2 gm. per c.c.) in water, what weight can he lift in air?

5. An article weighs 12·5 gm. in air, 8·5 gm. in methylated spirit of density 0·8 gm. per c.c. Calculate density of the article.

6. Find the relative density of a piece of silver which weighs 84·65 gm. in air, 81·35 gm. in water.

7. A piece of iron (density 7·2 gm. per c.c.) weighs 47·16 gm. in air. What will it weigh in water?

8. What will a glass cube, side 4 cm. long, weigh in air and in water if the density of glass is 2·5 gm. per c.c.?

9. A marble weighs 4·5 gm. in water and 2·1 gm. in sulphuric acid of density 1·8 gm. per c.c. What is the volume of the marble?

10. What weight of marble (relative density 2·7) can a boy support in water if 119 lbs. is the greatest weight he can lift in air?

11. A piece of tin, volume 11·5 c.c., weighs 83·95 gm. Calculate its density.

12. A piece of rock salt weighs 9·45 gm. in air and 4·41 gm. in brine (density 1·12 gm. per c.c.). What is the density of rock salt?

13. A lump of salt of density 2·1 gm. weighs 16·8 gm. in air. What will it weigh in methylated spirit of density 0·82 gm. per c.c.?

14. An ebony cylinder of height 5 cm. weighs 9·24 gm. in air and 1·54 gm. in water. What is its diameter?

CHAPTER IX.

FLOATING BODIES AND HYDROMETERS.

52. Reasons for floating or sinking.

In Fig. 57 a body is shown immersed in a liquid.

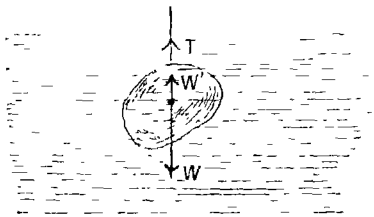


Fig. 57.

Let W represent the weight of the body,
and W' the upthrust due to the displaced liquid.

These two forces act in opposite directions in the same straight line.

(1) If $W > W'$, the body will sink, unless it is supported by a string. The pull on the string, *i.e.* the tension $T, = W - W'$.

(2) If $W = W'$, the body will remain wherever it is placed in the liquid, the density of the solid being equal to that of the liquid.

(3) If $W < W'$, the body will rise to the surface and float: the volume of the body under the surface then displaces a weight of water equal to the weight of the body, *i.e.*

Weight of a floating body = weight of liquid displaced.

*53. (1) To find the Density of Wood by Flotation.

Let D be the **Density** of a smooth *rectangular* block of wood (Fig. 58). Smear the block with vaseline and rub it dry.

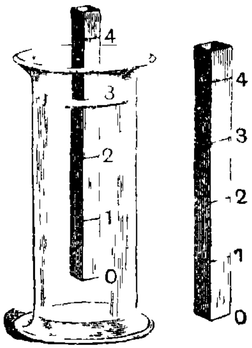


Fig. 58.

Since the block is rectangular its area of cross-section is constant $= A$ sq. cm.

Let

the length of the block = (say) **45 cm.,**

then

the volume of the block $= A \times 45$ c.c.,

and

the weight of the block

$$= A \times 45 \times D \text{ grams} \ldots (1).$$

Float the block in water, gently supporting it vertically.

Measure the **depth** to which it sinks = (say) 30 cm.

Then the volume of water displaced $= A \times 30$ c.c.

$\therefore$ the weight of the block $= A \times 30 \times d_w$ grams ....(2),

where d_w = density of water = 1,

but (1) = (2),

$$\therefore \; A \times 45 \times D = A \times 30 \times 1 \quad \ldots\ldots\ldots\ldots\ldots (3),$$

$$\therefore \; D = \tfrac{30}{45} = 0{\cdot}6,$$

or, in words,

$$\text{the } \textit{density of the wood} = \frac{\text{depth to which the block sinks in water}}{\text{total length of rectangular block}}.$$

*(2) To find the Density of a Liquid by the Flotation Method.

Let d_s be the **Density** of the liquid (say *Methylated Spirit*). Float the same block as was used in the above experiment, and measure the vertical **depth** to which it sinks = (say) **37 cm.,**

then the weight of the block $= A \times 37 \times d_s$ grams,

but by (2) " " " $= A \times 30 \times d_w$.

$$\therefore\ A \times 37 \times d_s = A \times 30 \times d_w \quad \ldots\ldots\ldots\ldots\ldots\ldots(4),$$

$$\therefore\ 37 \times d_s = \quad 30 \times d_w = 30 \times 1 \ldots\ldots\ldots\ldots(5),$$

$\therefore$ the Density of Methylated Spirit $= d_s = \frac{30}{37} = 0{\cdot}81$.

Let us put into words Equation (5), remembering that we are using a block of uniform cross-section and that we are floating it first in spirit and then in water. Equation (5) then reads:

depth of sinking in spirit × density of spirit

= depth of sinking in water × density of water,

$$\therefore\ \frac{\text{density of spirit}}{\text{density of water}} = \frac{\text{depth of sinking in water}}{\text{depth of sinking in spirit}},$$

i.e. the densities of the two liquids are inversely proportional to the depths to which a block of uniform cross-section will sink in those liquids.

54. A **hydrometer** is a graduated float used for finding the relative densities of liquids by comparing the depths to which the instrument sinks in the various liquids.

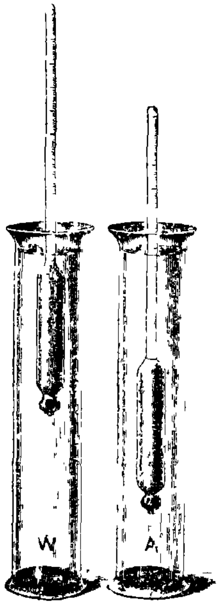
The simplest form of hydrometer has already been constructed in Exp. B, § 43 (see Fig. 51 *b*).

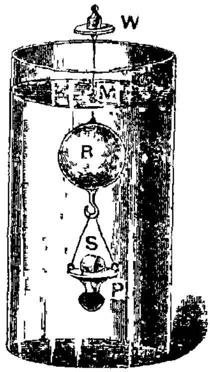
***Exp. i. Float the loaded test-tube** (Fig. 51 *b*) in various liquids (water, salt solution, methylated spirit, petrol, glycerine) and note the depths to which it sinks. Find the *relative density* of each liquid by dividing the depth to which the hydrometer sinks in water by the depth to which it sinks in the liquid. (Cf. § 53 (2).)

***Exp. ii.** Confirm your results in Exp. i by using the **common hydrometer.**

The **common hydrometer** (Fig. 59) is constructed on the same principle. It is made of glass and consists of a bulb (B) containing mercury, a float (F) containing air and a stem (S) which carries a graduated scale. When the hydrometer floats in the liquid the *specific gravity* may be read on the scale by noting the graduation which is level with the surface. Since the cross-section of the instrument varies the graduations are not equal.

Fig. 60 shows the same hydrometer floating in water and in alcohol.





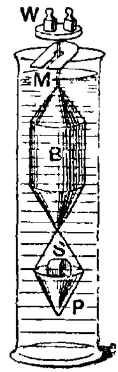


Fig. 59. Fig. 60. Fig. 61 *a*. Fig. 61 *b*.

55. Nicholson's Hydrometer (Figs. 61 *a* and *b*) consists of a hollow cylinder or bulb (*B*) which supports a heavy pan (*P*) below and a fine stem above, on which is a mark (*M*), bearing a lighter pan (*W*).

***Exp. i. To find the Rel. Density of a Solid** (say a *stone*). Float the Nicholson's hydrometer in sufficient water to cover it about ½ inch above *M*.

Always adjust the weights until *M* is in the surface of the water.

(1) Add weights to *W* =(say) 12 grams.

(2) Put the *stone* and *weights* in *W*. The *weights* alone = (say) 3 grams.

∴ Weight of *stone* = 9 grams.

Remove the weights, lift the hydrometer and place the stone (S) in the lower pan (P).

(3) Add weights to W until M is again in the surface = (say) 6·39 grams.

$\therefore$ the upthrust = weight of water displaced by the stone

$= 6{\cdot}39 - 3 = 3{\cdot}39$ grams.

$$\therefore \text{ the Rel. Density of the stone} = \frac{9 \text{ grams}}{3{\cdot}39 \text{ grams}} = 2{\cdot}65.$$

If the solid is less dense than water (say a cork), a piece of fine wire for tying the cork to P should be attached to P throughout the experiment.

***Exp. ii. To find the Rel. Density of a Liquid** (say *salt solution*) weigh the Nicholson's hydrometer and add weights to W to sink the instrument to the mark M when it is floating (1) in salt solution, (2) in water. Let the hydrometer weigh 200 grams.

(1) Weights added to W when hydrometer is sunk to mark M in salt solution = 54·5 grams.

$\therefore$ Weight of salt solution displaced $= 200 + 54{\cdot}5$ grams.

(2) Weights added to W when hydrometer is sunk to mark M in water = 12 grams.

$\therefore$ Weight of an equal vol. of water $= 200 + 12$ grams.

$$\therefore \text{ Rel. Density of salt solution} = \frac{254{\cdot}5}{212} = 1{\cdot}2.$$

Examples IX (Floating Bodies).

1. When a piece of elm wood of relative density 0·57 and volume 10 c.c. is floating in water, what volume will be immersed? What weight must be placed on it to just sink it?

2. A cube of white pine wood, 5 cm. side, floats in water with 2·55 cm. of each side immersed. Calculate the relative density of the wood.

3. If a piece of iron of relative density 7·6 and volume 6·8 c.c. floats in mercury of relative density 13·6, what fraction of its volume will be immersed?

4. An iceberg floats in sea water of relative density 1·025 with 500 cu. yd. above the surface. If the relative density of ice is 0·918, what is the total volume of the iceberg?

5. A loaded test-tube sinks to a depth of 12·3 cm. in water. To what depth will it sink in methylated spirit of relative density 0·82? If depth in turpentine is 14 cm., what is the relative density of turpentine?

6. A cork, relative density 0·25, is dropped into water in a graduated cylinder and causes the level to rise from 30 c.c. to 32 c.c. What is the volume of the cork?

7. What must be the volume of the chambers of a floating dock intended to support a vessel of 20,000 tons?

8. If a load of 135 tons is placed in the centre of a floating raft of length 60 ft. and width 40 ft., how much will the raft sink?

9. A loaded test-tube sinks 16·8 cm. in water, 15 cm. in brine. What is the density of the brine? How deep would it sink in milk of relative density 1·02?

10. 18·5 gm. are needed to sink Nicholson's hydrometer to the mark With a piece of ebony in upper pan 10·1 gm., and with ebony in lower pan 17·1 gm. are required. Calculate the relative density of ebony.

11. 16·19 gm sink Nicholson's hydrometer to the mark: with a piece of copper in upper pan 12·185 gm., with copper in lower pan 12·635 gm. are required. What is the relative density of copper?

12. When a glass stopper weighing 4 gm. and of relative density 2·5 is placed in the upper pan, 8·5 gm. are required to sink hydrometer to the mark. What weight will be needed when the glass is placed in the lower pan?

CHAPTER X.

ATMOSPHERIC PRESSURE.

56. The atmosphere is the envelope of air encircling the earth. It extends to a height of more than 100 miles. We have learnt (§ 40) that air in motion (wind) exerts pressure. We should therefore expect that air has weight. Galileo (b. 1564) proved this by showing that a copper sphere increased in weight when air was pumped into it. We can easily show that a glass globe (Fig. 62) loses weight as air is pumped out. A litre flask rendered vacuous loses 1·293 grams in weight, if originally filled with dry air at 0° C under average atmospheric conditions of pressure.

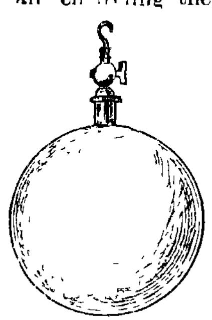


Fig. 62.

Exp. To find the weight of a litre of air.

Boil about 30 c.c. of water in a 300 c.c. round bottomed flask, fitted with a rubber cork through which a glass exit-tube passes. Attach a piece of rubber tube and a clip to the exit-tube (Fig. 63). When the water has boiled for 2 minutes it may be assumed that all air in the flask has been displaced by steam. Close the rubber tube with the clip and *at the same time* remove the flame. When the flask is cold, *weigh it* (say 87·21 grams). Open the clip; air rushes in; weigh again (say 87·58 grams). The difference between these weights (0·37 grams) is the *weight* of the *air* entering the flask. Measure the volume of water remaining in the flask (say 28 c.c.) and also the total capacity of the flask (say 334 c.c.). The difference (306 c.c.) is the *volume of air* entering the flask. Therefore 306 c.c. of air weigh 0·37 grams.

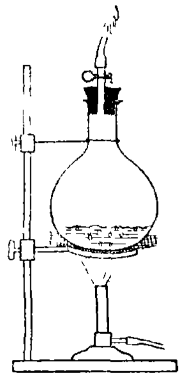


Fig. 63.

$$\therefore\ 1000 \text{ c.c. of air weigh } \frac{0\cdot37 \times 1000}{306} = 1\cdot21 \text{ grams}^{1}.$$

57. Air exerts pressure.

Exp. i. Tie a piece of rubber sheeting (*e.g.* football bladder) over the rim of an open cylinder, ground at one end (Fig. 64). Place the cylinder on the well-greased plate of a Tate's air pump and exhaust the contained air. As the air is removed the pressure of the atmosphere is shown by the collapse and final bursting of the rubber sheet.

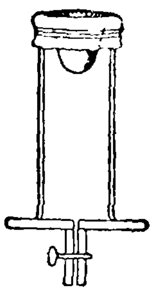


Fig. 64.

Fig. 65.

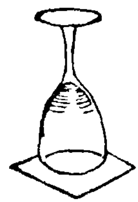


Fig. 66.

Exp. ii Von Guericke's (Magdeburg) Hemispheres. This experiment was originally shown by Von Guericke at Magdeburg about 1650. Two tightly fitting hemispheres (Fig. 65) are exhausted by an air pump. The tap is turned off and a handle is screwed on to the nozzle. Great force is now required to separate the hemispheres which enclose a vacuum.

Exp. iii. The "wine-glass" experiment demonstrates that air presses in all directions. Fill a wine-glass with water. Place a card on the top and, pressing gently with the hand to keep the card in position, invert the glass as shown in Fig. 66. On removing the hand, the card remains pressed against the rim and the water is retained inside the wine-glass. It is evident that the pressure of air from without is greater than the pressure of water from within. It is conceivable that we might have used a cylinder of

[1] This result gives the weight of a litre of air under the laboratory conditions of temperature (15° C.) and pressure (763 mm.) at the time of the experiment.

such length that the pressure of the water inside would have been greater than the atmospheric pressure: the water would then have escaped on inverting the cylinder.

Exp. iv. Punch a round hole in the lid of a mustard canister. Select a small cork to fit the hole. Solder the lid on the canister, making the latter completely airtight. Boil a little water in the canister and when it is boiling put the cork tightly in the hole and remove the flame. Quickly condense the steam inside the tin by pouring cold water on the outside. A vacuum is formed and the pressure of the air causes the tin to collapse and crumple up

Exp. v. An elongated U-tube about a yard in length has the end of one arm fitted with a tap. The tap is opened and mercury is poured in by the funnel on the left until the tube is half full. The mercury is at the same level on both sides (L_1), showing that the pressure is the same, both sides being open to the atmosphere. By connecting to a Bunsen's vacuum pump, air is now pumped out of the right-hand arm and consequently the pressure is gradually lessened until an almost perfect vacuum is obtained. The mercury rises on the right but falls on the side open to the atmosphere. The difference in level ($L_2 L_3$) is measured and found to be about 30 inches or

$$760 \text{ mm.} = H.$$

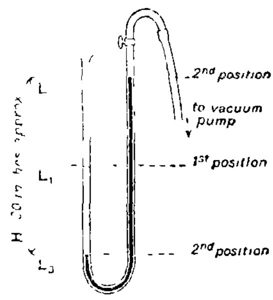


Fig. 67 *a*.

The tube is next disconnected from the vacuum pump when the mercury regains its original position. The tap being still *open* the tube is tilted (Fig. 67 *b*) until the mercury reaches the tap (Fig. 67 *c*), when the latter is immediately *closed* and the tube replaced by the vertical position. A space, a vacuum, now appears between the tap and the mercury and the difference in level of the mercury on the two sides is found to be about 30 inches as before (Fig. 67 *a*).

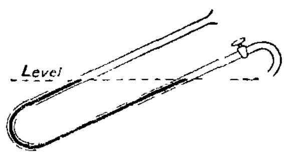


Fig. 67 *b*.

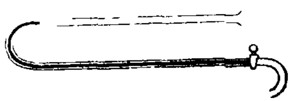


Fig. 67 *a*.

Exp. vi. (*a*) Dip the end of a tube about a yard long vertically in water and create a partial vacuum by sucking the air out at the upper end. The water rises in the tube. Explain this.

(*b*) Repeat the experiment, but dip the long tube vertically in mercury (density 13·6); it is impossible to suck the mercury up the tube more than a few inches. Explain this.

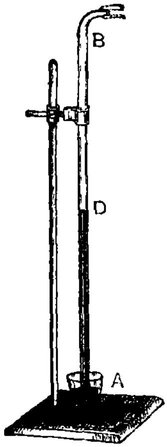


Fig. 68.

(*c*) Connect the tube to a Bunsen's vacuum pump with a piece of pressure tubing and exhaust the air from the tube (Fig. 68). The mercury rises gradually until the vertical height of the column AB = nearly 30 inches = H.

Exp. vii. (*a*) Fill a tube of about a yard in length, closed at one end, with *water* (coloured). Place the thumb tightly over the open end, thus enclosing the water: invert the tube in a vertical position in a bowl of water and remove the thumb. The tube remains full of water.

(*b*) Using a similar clean tube, nearly fill it with mercury. Close it with the thumb and, by tilting, allow the bubble of air to run up and down the tube several times to remove smaller air bubbles. Next, completely fill the tube with mercury, close the end with the thumb and open under a bowl of mercury, with the tube vertical (Fig. 69). The mercury falls until a space (the **Torricellian Vacuum**) appears with its lower limit about 30 inches above the level of the mercury in the trough. Slant the tube and notice that the upper level of the mercury remains constant until there is no vacuum space in the upper part of the tube. This constitutes a mercury **barometer**. The average vertical height of the column = 760 mm. or about 30 inches.

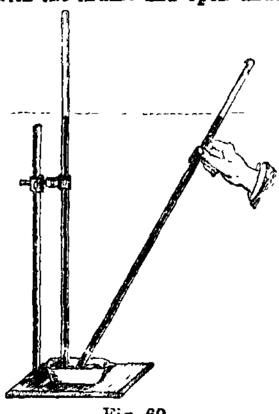


Fig. 69.

(*c*) Place the bowl of mercury and barometric tube on the plate of an air pump (Fig. 70). Place a greased bell jar over the tube and fit a perforated rubber cork (previously prepared and greased) over the tube: gradually work the cork into position until the air space over the bowl is completely closed. Now exhaust the air by the pump. Explain the reason for the gradual fall of the mercury in the tube. If a perfect vacuum could be obtained the mercury within and without the tube would be at the same level.

58. Fortin's Barometer.

In obtaining the height of the barometric column (H) the difference of level between the mercury in the trough and in the tube is measured When the pressure is lessened the column sinks and more mercury flows into the trough, thereby raising the level in the trough. In Fortin's Barometer (Fig 71) the scale

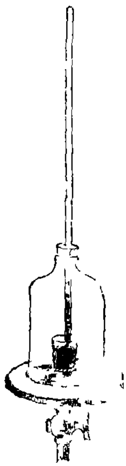


Fig. 70.

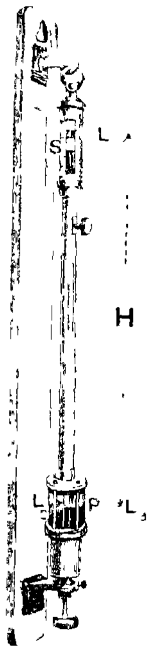


Fig. 71.

(S) is attached to the tube, the zero of the scale being at the end of a fixed pin (P). The level of the mercury ($L_2 L_3$) is always adjusted to touch the point P by altering the capacity of the trough or cistern, which is fitted with a leather base moveable by means of a screw below.

Aneroid Barometer. This instrument is constructed on the principle that a thin metallic box, from which the air is partially exhausted, will expand or contract as the pressure of the air on the outside diminishes or increases. The box, "concertina-like," is cylindrical with corrugated sides; the top rises or falls with variations of the pressure. To the top is attached a lever, the end of which exaggerates the movement. From this end a minute chain passes round the axis of a pointer, the chain being kept taut by tension on a spiral spring.

General Remarks on Barometers.

The height of the barometer column measures the pressure of the atmosphere, which averages about 15 *lbs. per sq. in.*

If water were substituted for mercury in a barometer, a tube of more than 34 feet in length would be needed. For, since the density of mercury = 13·6, the weight of the atmosphere would balance a column of *water* 30 in. × 13·6 = about 34 ft. Such a column of water one square inch in section would weigh

$$\frac{62{\cdot}5 \text{ lbs.}^{1}}{1728} \times 30 \times 13{\cdot}6 = 14{\cdot}7 \text{ lbs. (approx.)}.$$

A glycerine barometer has a column of height = 27 ft.

Mercury is most useful for barometers because (*a*) its density is high (13·6); (*b*) it does not wet glass; (*c*) it does not evaporate readily; (*d*) it expands equally for equal variations of temperature.

It is evident that if we ascend or descend in the ocean of air around us there will correspondingly be less or more air above us; hence the barometer column will fall if we carry the instrument up a mountain or ascend with it in a balloon, but if we descend the mercury rises in the tube, the variation for low altitudes being approximately one inch in the height of the barometer column for a vertical rise or fall of 1000 ft.

[1] A cubic ft. of water weighs 1000 oz. = 62·5 lbs

*59. Density of a Liquid by Hare's Apparatus.

The relative densities of two liquids may be found on a similar principle to that shown in § 45, where two columns of liquids were balanced against each other. Hare's apparatus (Fig. 72) consists of two parallel vertical tubes, connected at the top by a three-way tube fitted with a tap T, dipping into beakers containing the two liquids (say *water* and *salt* solution). The liquids are drawn up the tubes to suitable heights by suction at T. The heights of the columns (h_1 and h_2) are carefully measured from the levels L_1 and L_2 respectively. For instance, suppose that—

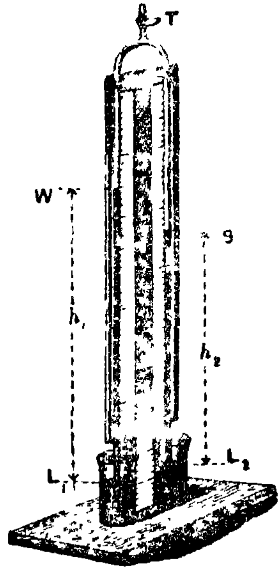


Fig. 72.

Height of column of water h_1
$= 18{\cdot}1$ cm.
Height of column of salt solution h_2
$= 15{\cdot}2$ cm.

Let D_1 = density of water = 1
and D_2 = density of salt solution.
Then, if

H = pressure (grams per sq. cm.) of the atmosphere on the liquid in each beaker,

and

p = pressure (grams per sq. cm.) of the air in tube connecting the columns,

the pressure at $L_1 = H = h_1 \times D_1$ grams per sq. cm. $+ p$
and the pressure at $L_2 = H = h_2 \times D_2$ grams per sq. cm. $+ p$.

$$\therefore\ h_1 \times D_1 = h_2 \times D_2,$$
$$\therefore\ 18{\cdot}1 \times 1 = 15{\cdot}2 \times D_2,$$

$$\therefore\ D_2 = \frac{18 \cdot 1}{15 \cdot 2} = 1 \cdot 19 \text{ grams per c.c.}$$

Cf. result of § 45, Exp. i. $\frac{D_1}{D_2} = \frac{h_2}{h_1}$.

60. Variation of the Density of the Air.

In § 36 we referred briefly to the constitution of a gas. The molecules of a gas are free to move, perfectly elastic and in constant motion. The gas completely fills the containing vessel. Its *density* is measured by the mass of molecules per unit volume and will vary as air is pumped into or pumped out of the vessel. At the same time the *pressure* will rise or fall, for the greater the number of molecules there are confined in a certain space the greater the number of bombardments of the molecules against the containing walls. This *variation of the density of the air with the pressure* may be shown as follows: In a bell jar connected

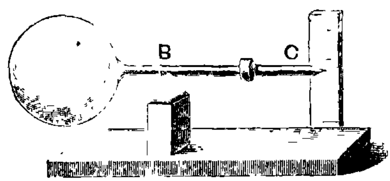


Fig. 73.

with an air pump, a closed bulb (Fig. 73), having a rod and moveable screw attached, is balanced at *B*. The air in the bell jar is then pumped out, its density lessening as the pressure diminishes. As the weight of the air displaced by the bulb is less, its buoyancy is also less and balance is disturbed, *C* rising: when air is again admitted the balance is restored as before.

The true weight of a body is its weight in vacuo, for its weight in air is less than its weight in vacuo by the weight of air displaced (see § 46). For instance the weight in air of 1000 c.c. of water at 4° C. is 1000 grams – weight of 1000 c.c. of air = 1000 – 1·2 grams (approx.).

61. Boyle's Law. The last experiment is a suitable

introduction to the subject of the variation of the *volume* of a given mass of gas with the *pressure.* If we refer to the "bicycle pump experiment" (§ 36), we shall remember that as the pressure on the air enclosed in the cylinder of the pump was increased the volume diminished, but on releasing the pressure the piston shot back, showing that the contained air has increased in volume If we assume a knowledge of the constitution of a gas, this result was to be expected, for if we confine a given number of molecules in a smaller space there will be a consequent increase in the number of bombardments on the walls of the vessel, *i.e.* an increase of pressure. Robert Boyle of Edinburgh in 1660 experimented somewhat similarly on what he called "the spring of the air."

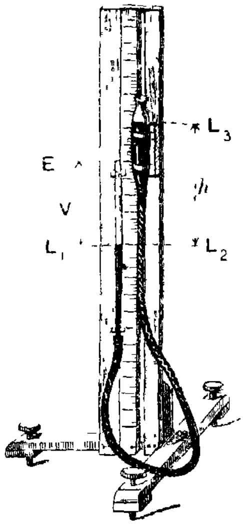


Fig. 74.

Boyle's Law.

The volume of a given mass of gas varies inversely[1] as the pressure, the temperature remaining constant.

Exp I. To verify this law we use the apparatus shown in Fig. 74. A small quantity of air is confined in a straight tube, the end of which (E) is closed; the other end is connected with a rubber tube filled with mercury which terminates in a glass reservoir which moves vertically

[1] **When quantities vary inversely their product is constant.** Thus if (say) 48 apples are to be divided equally among boys, the greater the number of boys the less the number of apples for each boy, or, the less the number of boys the greater the number of apples. If there were 48 boys each would get 1 apple, if 24 boys each would get 2 apples, if 16 boys each would get 3 apples, if 12 boys each would get 4 apples—the product of number of apples each boy gets and number of boys is **constant** (=48 in this case), the number of apples for each boy is said to *vary inversely* as the number of boys.

along a cm. scale and is open to the air. The **volume** of the enclosed air (V) is measured along the scale (EL_1) and its **pressure** (P) is obtained by adding the difference in level (h) of the mercury in the two limbs to the atmospheric pressure H, both h and H being expressed in the same units (say cm. of mercury pressure). If the level L_3 is *above* the level L_1L_2, then h is positive, if *below* then h is negative.

The observations and results should be entered as follows:—

H = atmospheric pressure = (say) 76 cm.

Level of E on scale = (say) 55 cm.

Level L_3 on scale	Level L_1L_2 on scale	$P=H+h$ = Pressure in cm. of Mercury	V = Volume in cm. along tube	PV = constant
92	55	76 + 37	80 − 55	113 × 25 = 2825
50	45	76 + 5	80 − 45	81 × 35 = 2835
22	35	76 + (− 13)	80 − 35	63 × 45 = 2835
10	30	76 + (− 20)	80 − 30	56 × 50 = 2800

It is advisable to take the observations in the first and second columns *as quickly as possible* as the temperature may vary if there is delay: the results may be worked out afterwards.

Plot these results on squared paper, taking the pressures as ordinates. The curve obtained is called a *rectangular hyperbola*.

Exp. ii. If the apparatus described in Exp. i is not available, the following simple method will suffice for volumes measured at pressures below atmospheric pressure (H). Half fill with mercury a straight barometer tube clamped vertically (Fig. 75). Accurately measure along the tube the volume (V_1) of air between the mercury and the open end of the tube. V_1 will be at a pressure $H = P_1$. Closing the end of the tube with the thumb, invert it in a glass cylinder containing mercury; remove the thumb and measure the volume of enclosed air (V_2) and the height of the mercury column (h) when the tube is again clamped vertically. The pressure of the enclosed air (P_2) = $H - h$. It will be found that

$$P_1V_1 = P_2V_2 \text{ (approximately).}$$

Keeping the lower end in the mercury, raise and lower the tube and take fresh readings of V and h: work out the product PV as before.

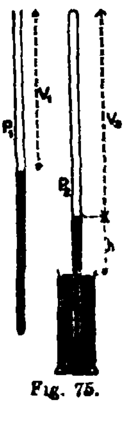


Fig. 75.

62. To find the Volume which a definite weight of a gas occupies at a changed Pressure, i.e. *to correct the volume when the pressure changes.*

We have learnt that for a given mass of gas

its Pressure × its Volume = a constant. [Boyle's Law.]

Suppose that we have 1000 c.c. (V_1) of gas at 700 mm. pressure (P_1), then the constant quantity = 700 × 1000 in this particular case.

If the pressure now changes to 800 mm. (P_2)

the changed pressure (P_2) × the new volume (V_2)

$$= 700 \times 1000,$$

$$\therefore\ 800 \times \text{the new Volume} = 700 \times 1000,$$

$$\therefore\ \text{the new Volume} = \frac{700 \times 1000}{800} = 875 \text{ c.c.}$$

This result may be obtained directly (1) by *substituting in the formula*

$$P_1 V_1 = P_2 V_2,$$

or, it may be obtained (2) by the *unitary method*, as follows :—

At 700 mm. pressure the volume of the gas is 1000 c.c.

$\therefore$ 1 mm. " " " " 1000 × 700 c.c.

$\therefore$ 800 mm " " " $\dfrac{1000 \times 700}{800} = 875$ c.c.

63. Revision Experiment.

To find the Density of Coal Gas relative to that of air Weigh a dry 500 c.c. flask, fitted with a cork. Fill the flask full of coal gas by displacing the air downwards, passing a rubber tube from the gas supply vertically upwards to the top of the inverted flask Remove the tube and quickly insert the cork, keeping the flask still inverted. Weigh the flask full of coal gas. Find the capacity of the flask. Calculate the weight of air it contains (1000 c.c weigh 1·2 grams) and hence find the weight of the vacuous flask By subtraction find the weight of the coal gas contained in the flask and hence calculate the weight of 1000 c.c. of coal gas.

EXAMPLES X (ATMOSPHERIC PRESSURE).

1. A 200 c.c. flask weighs 70·96 gm. when exhausted, 71·21 gm. when full of air. Calculate the density of air. What would the flask weigh if filled with oxygen (density 0·0014 gm. per c.c.)?

2. Calculate the weight of air in a room 8 m. long, 5 m. wide, 4 m. high, if the density of air is 0·00126 gm. per c.c.

3. All the air is driven out of a flask by boiling water in it. The clip is closed and the flask when cold weighs 62·72 gm. When air is admitted the weight becomes 62·875 gm. The volume of the water in the flask is 17 c.c.: total volume of flask 139 c.c. Find the weight of a litre of air.

4. A sealed glass globe of 1 litre capacity weighs 80 gm. in air of density 0·001293 gm. per c.c. What will it weigh in a vacuum?

5. A balloon contains 10 litres of hydrogen (density 0·00009 gm. per c.c.). If the density of air is 0·001293 gm. per c.c., what is the lifting power?

6. If in Hare's apparatus the height of the alcohol column is 43·3 cm and that of the water 35·5 cm., find the relative density of alcohol.

7. How high will the water column be in Hare's apparatus if the height of a column of alcohol of density 0·82 gm. is 35 cm.?

8. A bladder containing 25 c.c. of air at a pressure of 76 cm. is placed under the receiver of an air pump and the air is withdrawn till the pressure is 40 cm. Calculate the new volume of the bladder.

9. When the mercury is level in a Boyle's tube the air in the closed limb measures 22 cm., the barometer height being 76 cm. What will it measure when the mercury is 34 cm. above the common level? What will be the height of the mercury above the common level when the air measures 11·4 cm.?

10. A certain quantity of air measures 365 c.c. under normal pressure (76 cm.). What will it occupy under (i) 78 cm. pressure, (ii) 80 cm. pressure?

CHAPTER XI.

WATER- AND AIR-PUMPS, HYDRAULIC PRESS, SIPHON, DIVING BELL.

64. A garden **syringe** (Fig. 76) is familiar to all. If the nozzle (C) is placed under water and the solid piston drawn out by the handle, water runs up to fill the space in the cylinder (AB) which would otherwise be vacuous. Knowledge of Chap. X tells us that the pressure of the air on the surface of the water in the tank forces the liquid into the cylinder. If the liquid were *mercury* instead of water we know that if the cylinder of the syringe were longer than 30 inches and were held vertically it could not be completely filled with mercury but a vacuum would appear when the level of the mercury had been raised about 30 inches above the level of the tank. Similarly it would not be possible to completely fill a syringe with water by suction if the piston could move vertically in a cylinder of more than 34 feet in length.

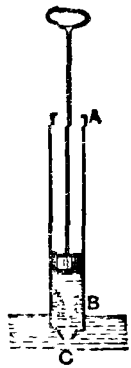


Fig. 76.

Common Suction Pump.

In Fig. 77 is shown a common suction pump which is merely a syringe with the nozzle (D) extended and fitted with a valve (C) or trap-door opening upwards; the piston (PP) is not solid but is

also fitted with valves (FF') opening upwards. When the piston (PP) is raised, water rises in D from the tank, the valve C preventing a back-rush of water. A few more strokes of the piston will fill the tube D, provided C is within 34 feet vertically of the level of the water D. Further pumping will raise more water into the cylinder (AB) and finally water will rush through the valves (FF') at the downstroke and be lifted and pass through E at the upstroke.

Lift Pump. If the outlet E (Fig. 77) were an upturned pipe fitted with a valve opening outwards and if the top (A) and collar of the pump were watertight, it would be possible to force the water up the tube E "by lifting." This arrangement is shown in Fig 78.

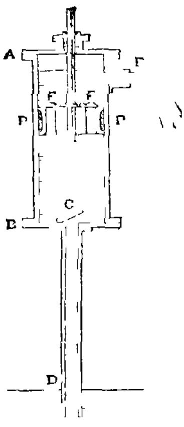


Fig. 77.

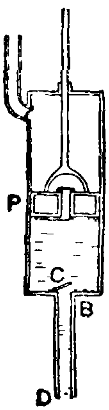


Fig. 78.

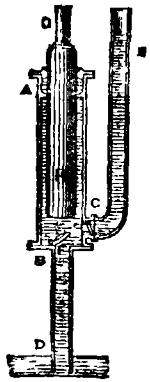


Fig. 79.

Force Pump. In Fig. 79 a similar suction pump is shown, but the piston (H) is solid (plunger) and the outlet tube E is fitted to the bottom of the cylinder. Both valves open in a direction away from the water supply. The valve F at the base of the cylinder must be within 34 feet vertically from the level of the water in the tank.

In **fire-engines** (Fig. 80) an air-dome (*A*) is inserted in the outlet tube where the elasticity of the compressed air maintains the pressure when the valve *C* is shut during the upstroke of *P*. Two pumps working with alternating strokes are used to keep a continuous supply of water passing into the chamber fitted with the air-dome.

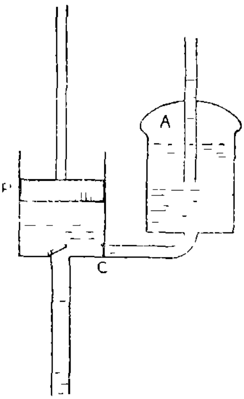


Fig. 80.

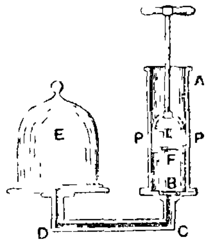


Fig. 81.

65. Air Vacuum Pump. This instrument is made on the principle of the common suction pump (*Fig.* 77), the vessel to be exhausted being attached to *D*. The arrangement is shown in Fig. 81, where *A* is the barrel of the pump, *PP* the piston, *F* and *B* the two valves both opening outwards, *DC* the connecting tube and *E* the vessel to be exhausted. The valves are of light construction and are in some instances ribbons of oiled silk lightly stretched across the end of the air passages. A perfect vacuum cannot be obtained by such an instrument because a limit is reached when the pressure of the air in the vessel is not sufficient to raise the valve.

Air Compression Pump.

Fig. 82 represents a pump suited for inflating footballs or compressing gas into cylinders. The valves at *E* and *F* both open inwards; the illustration shows the piston moving from

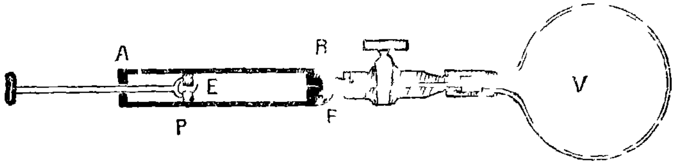


Fig 82

right to left, *F* being shut and *E* open; on pushing *P* to the right *E* shuts and the air contained is forced through *F* into the receiver *V*.

Sprengel's Vacuum Pump. An almost perfect vacuum may be obtained by using the apparatus shown in Fig 83. Mercury from a reservoir *E* is allowed to flow down a tube *ABC* dipping below the surface of the mercury in the trough *G*. The rate of flow is regulated by the tap *F*. A side tube *BD*, communicating with the vessel to be exhausted, is inserted at *B* which is more than 30 inches above the level of the mercury (*G*) in the trough. If the tubes *ABC* and *BD* were first filled with mercury and the taps *F* and *D* afterwards closed, the Torricellian Vacuum would appear in the arms *AB* and *DB*. If *F* and *D* are now opened, bubbles of air are drawn through *D* by the mercury as it drops down the tube *ABC* until finally a vacuum is obtained in the vessel attached at *D*.

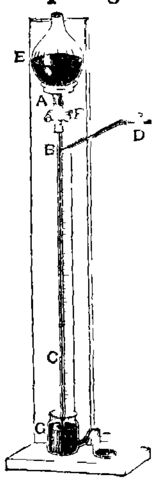


Fig. 83

Bunsen's Vacuum Pump and the *Steam Ejector* of the vacuum brake are constructed on the principle that water or steam rushing past an opening, which points in the

same direction as the stream of water or steam is passing, causes a suction of air through the opening.

66. Hydrostatic Bellows.

We have already learnt (§ 41) that in a liquid pressure is communicated throughout and in all directions In Fig. 84 bellows (A) communicate with a narrow vertical tube BC. The whole apparatus is supposed to be filled with water. A considerable weight may be placed on the bellows and be supported by the pressure of the water in the tube BC. Let the area of cross section of the tube be (say) 1 sq cm. The pressure at the level LL_1A will be equal to pressure due to the column of water in BC (say) 50 cm. in height above this level. The upward pressure on A will therefore be 50 grams per sq. cm , and if the area of A were (say) 700 sq. cm , the total upward pressure on A will be

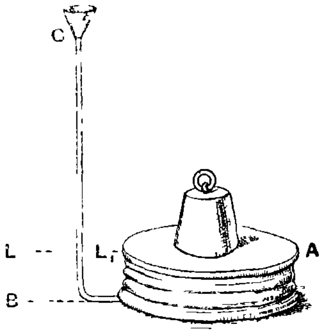


Fig. 84.

$$700 \times 50 = 35000 \text{ grams.}$$

Hydraulic Press. On this principle a hydraulic (Bramah's) press or a hydraulic lift is constructed. In the diagram (Fig 85) a force pump with plunger or piston (P) of small area of cross-section can at low pressure produce a great total pressure on piston (Q) of large area of cross section. Suppose, for instance, that the area of P is $\frac{1}{10}$ of the area of Q, then a force of F pounds on P will support a force (W) of $10F$ on Q.

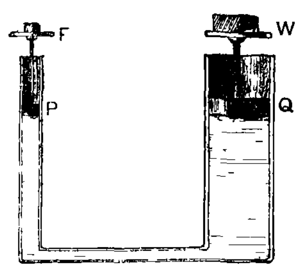


Fig 85.

If P has an area of a units and Q has an area of A units, then (assuming the pistons are weightless and frictionless) F will support a weight $(W) = \frac{A}{a} \times F$, i.e. $F \times A = W \times a$.

67. Exercise (in designing). Design and draw to scale an apparatus suitable for compressing bales of cotton at high pressure. Apply the principle indicated in the last paragraph and calculate from your drawing the total pressure exerted by the larger plunger, if the pressure of the water produced by the smaller plunger (which is part of a force pump) is 250 pounds per square inch.

68. Siphon. A siphon is a bent tube (Fig. 86) used for conveying liquids from one vessel to another by aid of atmospheric pressure. Usually *one arm* (BC) of the siphon is longer than the other. Having filled the tube with water and having closed the end of the longer arm with the finger, open the shorter arm under water. The head of liquid between L_2 and C causes the liquid to flow in the direction BC.

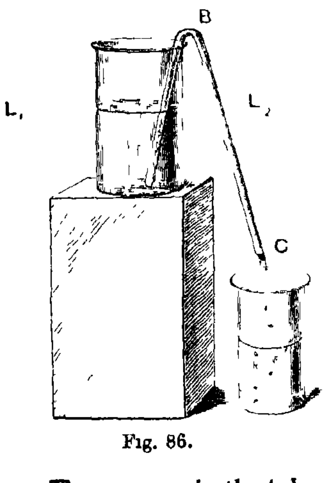


Fig. 86.

Let $L_1 L_2$ be the level of the surface of the liquid.

The pressure in the tube BC at L_2 = the pressure of the atmosphere (H).

The pressure in the tube BC at C
= H + the pressure downwards due to the liquid in the tube from L_2 to C.

∴ the liquid flows in the direction BC as long as C is below the level $L_1 L_2$.

69. The Diving Bell. If we take a tumbler and push it mouth downwards into a large beaker filled with water, we notice that air remains in the tumbler and that there is a slight rise of water in the tumbler as it is depressed further into the liquid. It is evident that the enclosed air is under a pressure which increases with the depth. In Fig. 87 is shown a diving bell

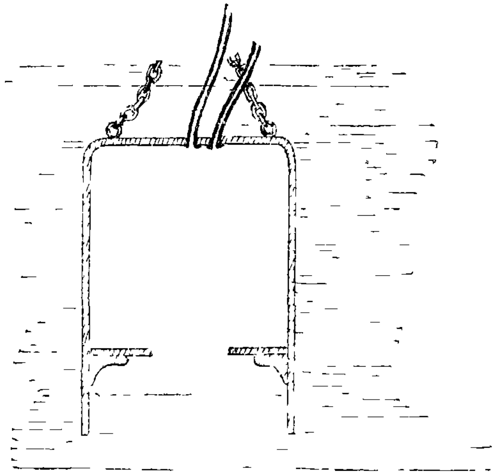


Fig. 87.

made on this principle. It is a large tumbler-shaped iron chamber, which may be lowered into water directly over some wreckage or a rock perhaps on which it may be necessary for men to work. Fresh air sufficient to cause free bubbling at the base must be pumped into the bell. Great discomfort is felt by men working at the high pressure which must be maintained to prevent water from entering the chamber from below even at moderate depths, but at depths up to 30 or 40 feet (*i.e.* at a pressure of 2 atmospheres) pain is only felt when the depth is changing.

EXAMPLES XI (MISCELLANEOUS QUESTIONS).

1. A piece of silver whose volume is 20 c.c. weighs 210 gm. The volume of a piece of aluminium weighing 2 Kgm. is 800 c.c. What are the densities of silver and of aluminium? What would you expect a solid block composed half of silver and half of aluminium to weigh if its total volume were 60 c.c.?
O. J.

2. An aluminium weight of 100 gm. is attached to a cork. What must be the weight of the cork in order that the combined aluminium and cork may float in water totally immersed? Relative density of aluminium = 2·5: cork = 0·24. O. J.

3. How much water must be added to 50 c.c. of alcohol of density 0·82 gm. per c.c. to make a mixture of density 0·9 gm. per c.c.?

4. A glass stopper weighs 10·25 gm. in air, 6·15 gm in water, and 6·97 gm. in methylated spirit. Calculate the relative density of glass and of spirit.

5. Find the relative density of pine-wood given that weight of wood alone = 4·08 gm.: weight of wood in air and sinker in water = 14·905 gm.: weight of sinker and wood in water = 6·405 gm.

6. 1 c.c. of lead (rel. density 11·4) and 21 c.c. of wood (rel. density 0·5) are fixed together. Show whether they will float or sink in water. O. J.

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7. A piece of sulphur weighs 12·2 gm. in air, 6·1 gm. in water and 6·71 gm. in oil. Calculate the relative density of sulphur and of oil.

8. Find the relative density of cork from the following: weight of cork alone = 0·89 gm.: weight of cork in air and sinker in water = 20·56 gm.: weight of cork and sinker in water = 17 gm.

9. A beaker of water is placed on the pan of a balance and counterpoised. A cube of copper of 2 cm. side is suspended from a separate support so that it is immersed in the water. What weight will be required to restore equilibrium and in which pan must it be placed?

10. If the hydrostatic bellows has a surface of 1 sq. ft., to what height must the tube (1 sq. in. cross-section) be filled with water in order to support a man weighing 10 stone?

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11. A metal cylinder 1 cm. in diameter and 15 cm. long weighs 122 gm. What is its density? Find to what extent it sinks or swims in mercury of density 13·6. O. J.

12. A piece of glass weighs 7·5 gm. in water and 8·1 gm. in turpentine of density 0·88 gm. per c.c. Find the volume and weight of the glass.

13. A Nicholson Hydrometer weighing 84 gm. requires 2·1 gm. to sink it to the mark in alcohol and 21 gm. when placed in water. Calculate the relative density of alcohol.

14. What is the pressure of the atmosphere in grams per sq. cm. when the barometer stands at 700 mm.? O. J.

15. A Bramah Press has two pistons of radii 2″ and 6″. If 10 lbs. be applied to the smaller piston, what pressure will the larger exert?

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16. If a cubic foot of water weighs 62·5 lbs. and a cubic inch of gold 0·698 lb., what is the specific gravity of gold? O. J.

17. If a mercury barometer registers 75 cm., what is the height of (*a*) a water barometer, (*b*) a glycerine barometer (density 1·25 gm. per c.c.)?

18. What weight of water fills a cylindrical tumbler of diameter 2·2 inches, height 4 inches? What is the pressure of the air on the surface of the water if the barometer stands at 30 inches?

19. In Hare's apparatus the height of a column of brine is found to be 35 cm. while the water column is 39·2 cm. Calculate the relative density of the brine.

20. To what depth must a diving bell 9 ft. high be sunk that the water may rise 3 ft. in it? (Barometer height $=30''$.)

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21. What will an ebony cylinder of height 7 cm. and diameter 8 cm. weigh in air and in water if the density of ebony is 1·2 gm. per c.c.?

22. What weight of lead $\frac{1}{12}$ inch thick is required to cover a hemispherical dome of diameter 11 ft.? (Density of lead $=11\cdot4$ gm. per c.c.)

23. A bottle weighs 115·91 gm. when full of water and 97·8 gm. when full of alcohol of density 0·8 gm. per c.c. Find the weight of the bottle.

24. 21·65 gm. are needed to sink a Nicholson Hydrometer to the mark in water. With a piece of cryolite in the upper pan 19·62 gm., with the cryolite in the lower pan 20·3 gm., are needed. Calculate the relative density of the cryolite.

25. If 5·6 lbs. applied to the smaller piston (diameter 4″) of a Bramah Press causes a pressure of 1 ton on the larger piston, what is the diameter of the latter?

REVISION QUESTIONS.

Paper A.

1. Write out the Metric Table of Length and give rough values for all the units. What was the metre originally intended to be?

2. Explain 'Parallax Error' by means of a diagram. How can it be avoided?

3. What is a Vernier? How would you make one to read to tenths of a millimetre?

4. State various methods for finding the circumference and diameter of a penny.

5. What is (*a*) a parallelogram, (*b*) a trapezium? How may their areas be determined?

6. Give sketches of apparatus used in the laboratory for measuring the volume of liquids.

7. What do you mean by 'Absolute Density,' 'Relative Density,' 'a litre,' 'Archimedes' Principle'?

8. Describe the process by which you would find the relative density of a piece of rock. O. J.

Paper B.

1. Write out the Metric Table of Area.

2. Give a sketch of a pair of Callipers and explain how they are used to find the external and internal diameter of a tube.

3. What precautions must be taken when measuring the diameter of a wire by means of a Screw Gauge?

4. How could you prove that the area of a triangle is

$$= \frac{\text{perpendicular height} \times \text{base}}{2}?$$

5. If the radius of a circle is r, what will the circumference be? What do you mean by π?

6. Give any methods you know for obtaining the volume of a key and mention the chief sources of error in each method. C. J.

7. Explain fully how the relative density of a liquid may be found by the relative density bottle. Why is there a passage in the stopper?

8. Describe some experiment to prove Archimedes' Principle.

PAPER C.

1. Write out the Metric Table of Volume.

2. Sketch and describe the Opisometer and explain how it is used.

3. How would you make a Wedge from squared paper by which the internal diameter of a glass tube may be found?

4. By what means can the length of a spiral coil of wire be found without unwinding it?

5. Suggest methods for finding the area (*a*) of an oak leaf, (*b*) of a field. What unit would you employ in each case?

6. How may the relative density of a liquid be found by means of a U tube?

7. If the relative density of iron is 7·2, why can an ironclad ship float?

8. How would you determine the relative density of a piece of Rock Salt?

PAPER D.

1. Write out the Metric Table of Weight.

2. How could the length of a racing track be found by means of a bicycle with a white patch on the tyre?

3. Sketch and describe a Spherometer and explain how it is used.

4. How could you prove that the area of a circle is πr^2.

5. What is 'Hare's Apparatus'? How is it used for finding the relative density of a liquid?

6. Describe some experiment to prove that the pressure of a liquid on a surface immersed in it depends upon the depth.

7. Distinguish between the *Mass* of a body and its *Weight*, and explain how each is measured. At what part of the earth's surface will a mass of 10 stone have the greatest weight? Give reasons for your answer. O. J.

8. Under what circumstances does a solid float in a liquid? A cubical box measuring 3 cm. each way just floats in water with one face level with the surface. How heavy is it? Explain exactly how you get your result. What will happen if the water is warmed slowly? O. J.

PAPER E.

1. What is the area of a Sphere? By what experiment could you verify your answer?

2. How may it be shown that the area of the curved surface of a cone is = ½ circumference × slant height?

3. Sketch a Balance and name the various parts, explaining the use of each.

4. Describe the construction and explain the use of a simple mercurial barometer. O. J.

How would you prove that the space above the mercury is a vacuum?

5. Explain how you would find, by the Principle of Archimedes, the relative density (*a*) of a solid lighter than water, (*b*) of a liquid. O. J.

6. What do you mean by a fluid? State the distinction between solids, liquids and gases.

7. How is it shown experimentally that if a pressure of any amount is applied to the surface of a liquid the liquid transmits this pressure to every surface in contact with it? O. J.

8. Explain, with diagrams, the action of 'Suction' and 'Force' Pumps. Why can water only be raised about 30 ft. with an ordinary pump?

PAPER F.

1. How may the weight of a litre of air be determined experimentally?

2. Sketch an arrangement by which you would show that the intensity of pressure is the same at all points at the same depth in water. O. J.

3. Describe with sketch and brief explanation an 'exhausting' air pump. O. J.

How would you use it to prove that the mercury in a barometer is supported by the Atmospheric Pressure?

4. In a beaker containing water floats a sphere of wood. If the vessel is put inside the receiver of an air pump what will happen to the wood (*a*) if the air is exhausted, (*b*) if more air is forced in? Give reasons.

5. Explain the action of a Siphon.

6. State the 'Principle of Archimedes' and apply it to a balloon. Explain why a balloon first rises rapidly and finally ceases to rise.

7. Describe Nicholson's Hydrometer. How is it used for finding the relative density (*a*) of a solid, (*b*) of a liquid?

8. Give a diagram of an Aneroid Barometer and explain its action. What is the meaning of the word 'aneroid'?

Paper G.

1. What is meant by (*a*) a sidereal day, (*b*) a mean solar day, (*c*) a second?

2. How would you prove that the time of oscillation of a pendulum is independent of the 'amplitude' of vibration when the amplitude is small?

3. Sketch and describe Fortin's Barometer.

4. The relation between the length of a pendulum (l) and the time of a complete oscillation 'to and fro' (t) is given by the equation

$$t = 2\pi\sqrt{\frac{l}{g}},$$

where $g = 981$ when l is measured in cms.,
$= 32$,, l ,, ft.

If the lengths of two pendulums are 100 cm. and 81 cm., compare the times taken by each in making 40 oscillations. Confirm your results by referring to the curve on p. 15.

5. Explain the action of Sprengel's Vacuum Pump.

SECTION III.

MECHANICS

CHAPTER XII.

VELOCITY, ACCELERATION, FORCE[1].

70. Motion. If two trains travelling in the same direction on parallel sets of rails are slowly running at the same speed into a station, they move like one train and each appears to passengers in the other train to be at rest. If the rails are smooth it is only possible to tell whether the trains are at rest by observing fixed objects on the station. "Rest" and "motion" therefore are relative terms. A cloud carried by the wind is at *rest* with regard to the air which carries it, but is in *motion* with reference to the ground over which it is travelling. Motion may be very complicated when several superimposed motions are imparted to a body. Consider for instance separately the various movements which are combined in the actual movement in space of a lamp hanging in the cabin of a rolling and pitching ocean liner; to the various motions of the vessel contending with tide and wind are added the displacements in space due to the earth's rotation and its motion round the sun.

Speed. We measure the speed of a moving body by the distance travelled in a given time. The units selected vary. Thus we speak of a train travelling at a speed of

[1] For the experimental treatment of Velocity, Acceleration and Force see § 75, which may be read before §§ 70—74 if preferred.

60 miles per hour, *i.e.* per 60 minutes,

or 1 mile per minute,

or $\frac{1}{60}$ mile per $\frac{1}{60}$ minute, *i.e.* per second,

or $\frac{1760 \times 3}{60}$ feet $= 88$ feet per second.

If the speed of a train moving 88 ft. per sec. is kept *uniform* for one hour, then the train would travel 60 miles in the hour.

71. Velocity. The word velocity includes a knowledge of the *direction* of a moving body at the particular moment of observation in addition to its speed.

Velocity is measured by *space traversed in a unit of time* and may be *variable* or *uniform*. Instances of uniform velocity are rare. An aeronaut for example might continue to fly 40 miles in each of 3 consecutive hours, but his velocity would from time to time be affected by the wind. His actual velocity at any instant would be measured by the space traversed in the particular second under observation, but this would not necessarily be the *same as his average velocity*, which would be the velocity with which he would traverse the whole distance in the same time with *uniform velocity*.

$$\text{His average velocity } (v) = \frac{\text{whole space traversed } (s)}{\text{time occupied } (t)}$$

$$= \frac{3 \times 40 \text{ miles}}{3 \text{ hours}}$$

$$= \frac{3 \times 40 \times 1760 \times 3 \text{ feet}}{3 \times 60 \times 60 \text{ sec.}} \quad 58\tfrac{2}{3} \text{ feet per sec.}$$

It is evident that

space traversed (s) = average velocity (v) × time (t),

i.e. $s = vt.$

Diagram of velocity, time and space.

Imagine a heavy stone to be moving over smooth ice with a *uniform* velocity of 8 feet per sec. How shall we represent graphically the velocity at any given instant, and the space traversed in any given time (say 4 secs.)? Draw on squared paper two axes OX and OY (§ 13) at right angles to

represent *time* and *velocity* respectively (Fig. 88). Measure from O along OY a distance OA (= 3 cm.) to represent a velocity of 3 ft./sec. Through A draw AB parallel to OX.

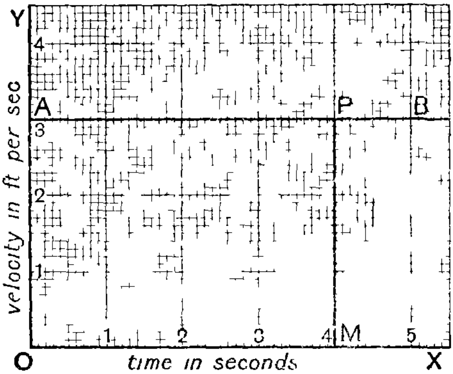

Fig 88

From O along OX measure OM (= 4 cm) to represent an interval of 4 secs Draw MP perpendicular to OX Then MP (= OA) represents the velocity (3 ft./sec) at the end of 4 secs The area $OAPM$ (= $OA \times OM$) represents the product of the time and the velocity and therefore also represents the space or distance traversed (= 12 feet) and the line AB is the velocity-time curve (§ 13) for the stone moving with a velocity of 3 ft./sec

Practical Example. Represent by means of a graph a uniform velocity of 5 ft. per sec and show how the space travelled in $2\frac{1}{2}$ secs can be found from it

72. Acceleration. In a diagram arranged on the same principle as that of § 71 the line AC (Fig 89) represents the time velocity curve of a body moving not with uniform velocity but with a velocity that uniformly increases at the rate of 2 units in every second **The rate of increase of velocity is called acceleration,** which in this example is 2 units per sec, *i.e.* 2 (ft per sec) per sec and is uniform

Through A draw AB parallel to OX, cutting the verticals (ordinates) PM, QN and RL in p q, r

Let us interpret the diagram (Fig 89)

The initial velocity is represented by $OA = u$ = (here) 1 ft per sec.,

the **acceleration** is represented by $pP - f$ = (here) 2 ft. per sec. per sec

	is represented by	symbol	number in diagram
The velocity at the beginning of 1st sec.	OA	u	1 ft./sec.
" " end " 1st sec.	$MP = Mp + pP$	$u + f$	3 ft./sec.
" " " " 2nd sec.	$NQ = Nq + qQ$	$u + 2f$	5 ft./sec.
" " " " 3rd sec.	$LR = Lr + rR$	$u + 3f$	7 ft./sec.
" " " " 4th sec.	$KC = KB + BC$	$u + 4f$	9 ft./sec.
∴ velocity at the end of the tth sec.	$= v =$	$u + ft$	

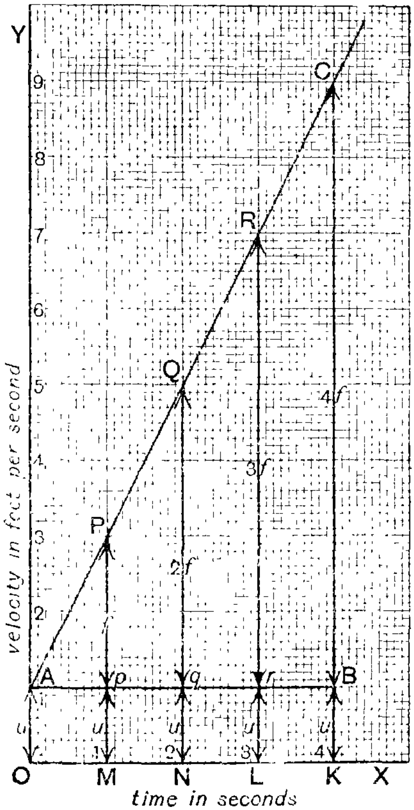

Fig. 89.

The *average velocity* for 1st sec. $=\dfrac{u+(u+f)}{2}=$ here $\dfrac{1+3}{2}=2$ ft./sec.[1]

$\therefore$ *distance* traversed in 1st sec. = av. vel. × time = here 2 ft., which is represented by

$$\frac{OA+MP}{2}\times OM = \text{area of } OAPM \ldots\ldots\ldots\ldots\ldots\ldots [\S\ 16\ (5)].$$

Similarly,

the *average velocity* for the 2nd sec. $=\dfrac{u+f+(u+2f)}{2}=$ (here) $\dfrac{3+5}{2}=4$ ft.

and the *distance* traversed for the 2nd sec. = (here) 4 ft.,

which is represented by $\dfrac{MP+NQ}{2}\times MN=$ area of $MPQN$ ....[§ 16 (5)],

and so on according to the following table:—

The particular second observed	Average velocity	Distance traversed = av. vel. × time	Represented graphically by area
1st	$\frac{1+3}{2}=2$ ft. sec.	$2\times 1=2$ ft.	$OAPM$
2nd	$\frac{3+5}{2}=4$ ft. sec.	$4\times 1=4$ ft.	$MPQN$
3rd	$\frac{5+7}{2}=6$ ft. sec.	$6\times 1=6$ ft.	$NQRL$
4th	$\frac{7+9}{2}=8$ ft. sec.	$8\times 1=8$ ft.	$LRCK$
For the whole time (t) observed	$\frac{u+(u+ft)}{2}$ $=u+\frac{1}{2}ft$	$(u\times\frac{1}{2}ft)\,t$ $=ut+\frac{1}{2}ft^2$	$OACK$

In diagram 89 the distance traversed (s) at the end of 4 seconds (t) is represented by the area $OACK$, but

$$\begin{aligned} \text{area of } OACK &= \text{rectangle } OABK + \text{triangle } ABC \\ &= \text{(here) } 1\times 4 \quad + \tfrac{1}{2}\,(2\times 4)\times 4. \end{aligned}$$

$\therefore$ **distance traversed in t sec. $= s = ut + \frac{1}{2}ft^2$.**

[1] It will be seen from the diagram when a body is moving with uniform acceleration that the average velocity for a given interval is half the sum of the velocities at the beginning and the end of that interval.

Exercise in Graphic Representation.

A body starts with a velocity of 5 cm. per sec. and moves for 5 secs. with a uniform acceleration of 2 cm. per sec. per sec. Find by means of a graph the space traversed in the third second and in the fifth second.

We have now obtained the formulae :--

$$v = u + ft \quad \ldots\ldots \quad \ldots \quad \ldots \quad \ldots\ldots\ldots(1),$$
$$s = ut + \tfrac{1}{2}ft^2 \ldots\ldots \ldots \ldots\ldots\ldots(2);$$

by squaring (1) we obtain

$$v^2 = u^2 + 2fut + f^2t^2$$
$$= u^2 + 2f(ut + \tfrac{1}{2}ft^2),$$
$$\therefore v^2 = u^2 + 2fs \quad \ldots\ldots\ldots\ldots\ldots\ldots(3).$$

Retardation. When a moving body gradually comes to rest, the rate at which its velocity diminishes is called its **retardation** which is equal but of *opposite sign* to the acceleration which must be imparted if the body started from rest and acquired the same velocity in an equal time. Hence a *retardation* is a **negative acceleration,** and the above formulae will hold good, but instead of f we must insert $-f$.

N.B. In the graph (Fig. 89) the **gradient,** $\frac{BC}{AB}$ or $\frac{8}{4} = 2$, measures the *acceleration*. Similarly in Fig. 88, the gradient being zero, the acceleration is zero.

73. Acceleration of a falling body (g). If a body is free to fall in vacuo it acquires an acceleration denoted by

g = **32 ft. per sec. per sec.** (approx.)[1]

= **981 cm. per sec. per sec.**

Example 1. A stone is thrown vertically *downwards* from a tower with a vel. of 10 ft./sec.; (*a*) how far will it travel in 4 secs.? (*b*) what will be its vel. when it has fallen 100 ft.? (*c*) what will be its vel. in 4 secs.?

(*a*) To find s, use $s = ut + \frac{1}{2}ft^2$,

(*b*) To find v, ,, $v^2 = u^2 + 2fs$,

(*c*) To find v, ,, $v = u + ft$,

remembering that $u = 10$ ft./sec., $t = 4$ secs.,

and $f = g = 32$ ft./sec.^{2}.

[1] Acceleration (ft. per sec. per sec.) is often expressed as ft./sec.^{2}.

Example 2. A stone is thrown vertically *upwards* with a velocity of 100 ft./sec.; (*d*) how high will it rise and (*e*) when will it come to rest?

(*d*) To find the height s, use $v^2 = u^2 + 2(-g)s$.

Remember that $v = 0$ and $u = 100$ ft./sec.

(when the stone is at its highest point $v = 0$).

(*e*) To find the time t, use $v = u + (-g)t$.

74. To find the value of "*g*."

The apparatus shown in Fig. 90 consists of a long pendulum bar AB oscillating about A; an adjustable heavy weight C may be moved along the bar to vary the time of vibration of the pendulum. A piece of thread is tied to a weight W and is passed over two pegs D and E and arranged to hold aside the bar. The edge of W almost touches the rod at its point of suspension. If the thread is burnt the pendulum and weight start at the same moment. The position of C must be adjusted so that the weight strikes the bar near B, *i.e.* at the end of one half-swing. Blacken the weight with the smoke of a lighted candle so that a mark is left on the rod; take the mean of three observations to obtain the distance s through which the weight falls before striking the rod. Measure s in *ft.* and *cms.* The weight falls a distance s while the pendulum is completing half a swing t. Count the time of 50 swings, remembering to begin counting 0, 1, 2, &c. and thus obtain the time of half a swing by dividing the whole period by 100.

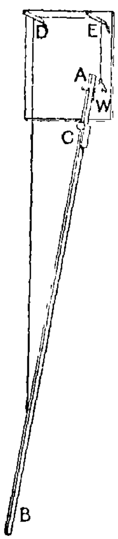

Fig. 90.

Calculate g in *ft./sec.*^{2} and in *cm./sec.*^{2}, using the formula

$$s = \tfrac{1}{2}gt^2.$$

75. Velocity, Acceleration and Force treated experimentally.

In the following experiments on **moving bodies** we have to observe (*a*) the space traversed (**length**), (*b*) the **time** occupied and (*c*) the **mass** moved.

We shall try to show that:—

(1) If a moving body is acted on by *no force* it travels over equal distances in a given time, *i.e.* it moves with **uniform velocity** (see § 35 and Exp. i *infra*).

(2) If a body is acted on by a force its velocity either increases or decreases (it does not remain constant) (see Exp. ii *infra*).

(3) If the *force* remains *constant* the rate of increase (or decrease) of velocity is *constant*; but if the **force** *increases or decreases, the rate of change of velocity*, or the **acceleration,** *increases or decreases* **in the same proportion** (see Exp. ii *infra*).

(4) If the *mass* of the moving body is altered, but the *force* causing motion remains *constant*, the *acceleration* changes in *inverse* proportion to the mass moved (see § 61, note and Exp. iv *infra*).

Exp. i. The body to be moved is a heavy wooden **trolley**[1] (Fig. 91), of

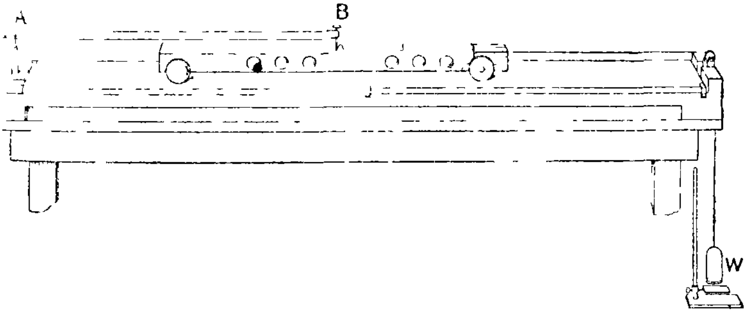

Fig. 91.

which the mass may be varied by putting metal cylinders into the six holes shown along its side. The wheels run smoothly, but on a level table the force of friction brings the trolley to rest. The table therefore must be *slightly tilted* so that the pull of the earth on the trolley will just balance the force of friction tending to stop the body. The trolley will then move as nearly as possible under the action of *no force*.

[1] The apparatus was invented by W. C. Fletcher, Esq., H.M. Chief Inspector of Secondary Schools.

As the time during which motion occurs is short, the method of measuring time by a swinging pendulum (§ 12) must be modified. A vibrating metal spring or bar (*AB*) is firmly clamped at *A*, while the free end *B* carries a finely pointed brush dipped in ink. When the free end is pulled aside and let go the bar vibrates and the brush moving backwards and forwards traces a wavy line on a card pinned on the moving trolley. As in the pendulum, the period of vibration is proportional to the square root of the length of the rod (*AB*) (cf. § 12). Adjust the length of the bar to obtain 5 complete 'to and fro' vibrations in one second; the period of a single 'to and fro' movement is therefore $\frac{1}{5}$ sec. and the time occupied by the trolley in moving through the distance between two adjacent crests of the wavy line is $\frac{1}{5}$ of a second.

A string attached to the trolley passes over a pulley and suspends a light pan containing weights (*W*). Pin a card on the left hand half of the trolley (see Fig. 91). Place the trolley so that it touches the upright at *A* and supports the pan (*W*) on the shelf of the retort stand, the string being taut. Set the bar vibrating and quickly lower the shelf about 4 inches. The trolley will move (1) through 4 inches under the action of a force (= weight of *W*, including pan), (2) through the remainder of its journey across the table,

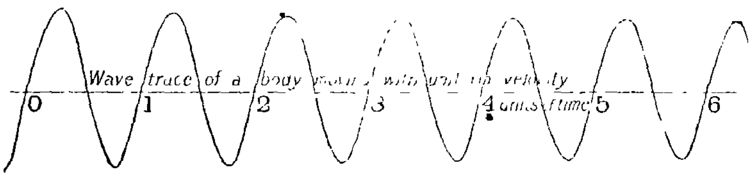

Fig. 92. ($\frac{1}{2}$ actual size.)

acted on by *no force*. Remove the card and draw a straight line (Fig. 92) along the length of the wavy line midway between crests and hollows. Measure the distances between alternate crossings of this straight line by the wave trace. *These distances are all equal* and were travelled over by the trolley in $\frac{1}{5}$ sec. each. The body therefore when acted on by *no force* moved with *uniform velocity*.

Exp. ii. To illustrate (2), p. 115, the scale pan and cord are removed. The table is then tilted and the trolley is (*a*) started from rest *down* the slope or (*b*) started by a gentle push *up* the slope. In the former case (*a*) it is evident that the velocity increases; while in the latter case (*b*) the velocity at starting may be adjusted so that the trolley comes to rest before reaching the top of the slope. Neglecting friction the only force acting on the body is the pull of the Earth which, as the table is not level, acts to some extent down the slope.

Exp. iii. To illustrate the third statement [(3), p. 115] the table is again levelled and the scale pan attached to the trolley. A wave-trace is taken

(Fig. 93) when the trolley starts from rest and is constantly pulled by the weight *W*. A second wave trace is obtained when the weights and scale pan are increased to 2*W*. In each case the rate of change of velocity (*acceleration*) is obtained as shown below and is found to be constant for each particular case, but the acceleration in the second case is double that in the first. Therefore, the mass of the body moved being kept constant, the *rate of change of velocity*, or acceleration, is *proportional to the force* causing motion.

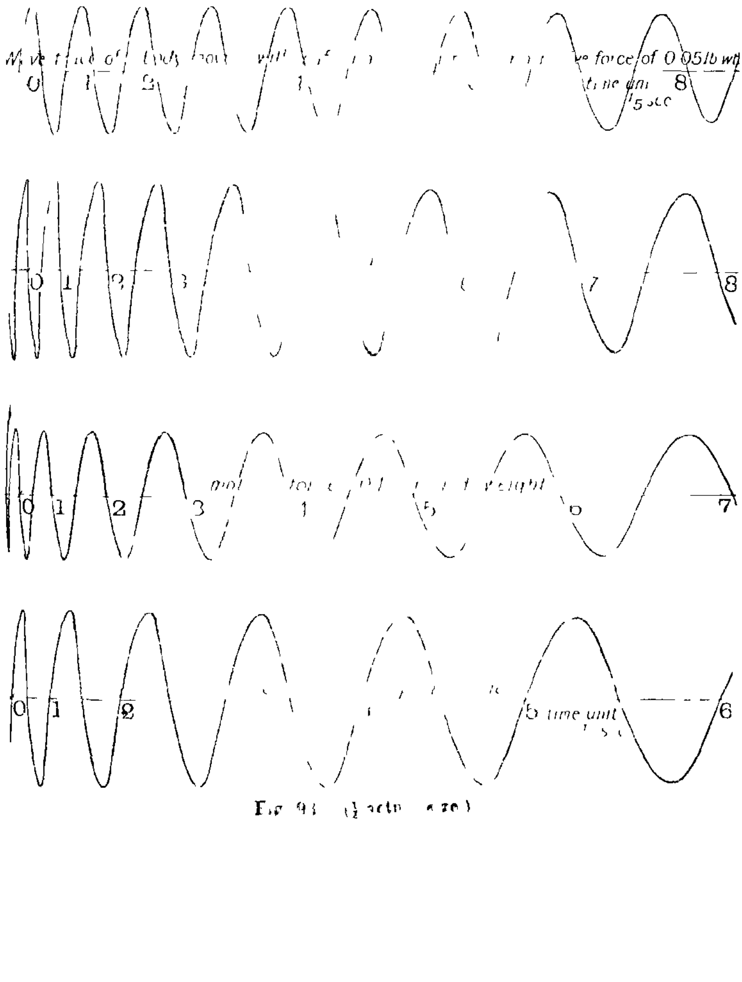

Fig. 93 (½ actual size)

Measurements from four traces by Fletcher's Trolley. Traces α – δ.

Time unit ⅕ sec.		Cms. travelled in ⅕ sec	Average vel. in cm./sec.	Increase of vel. per ⅕ sec.	Accelera-tion in cm./sec.^{2}	Motive force in lb. wt.
α	1	2·45	12·25			
	2	2·80	14·00	1·75 cm./sec.	8·75	0·05
	3	3·15	15·75	1·75	,,	
	4	3·50	17·50	1·75	,,	
	5	3·85	19·25	1·75	,,	
β	1	1·4	7·0			
	2	2·1	10·5	3·5 cm./sec.	17·5	0·10
	3	2·8	14·0	3·5	,,	
	4	3·5	17·5	3·5	,,	
	5	4·2	21·0	3·5	,,	
γ	1	1·55	7·75			
	2	2·60	13·00	5·25 cm./sec.	26·25	0·15
	3	3·65	18·25	5·25	,,	
	4	4·70	23·50	5·25	,,	
	5	5·75	28·75	5·25	,,	
δ	1	1·8	9·0			
	2	3·2	15·0	7·0 cm./sec.	35·0	0·20
	3	4·6	23·0	7·0	,,	
	4	6·0	30·0	7·0	,,	
	5	7·4	37·0	7·0	,,	

The numbers in the two last columns are in proportion, *i.e. acceleration* is proportional to *force*, when *mass* is constant.

Notice that *acceleration* is *constant* as long as *force* is *constant*.

Exp. iv. To illustrate (4), p. 115, the weights in the pan are not altered, *i.e.* the force causing motion remains constant, but wave-traces are obtained (1) when the trolley is loaded with all the metal cylinders, (2) after the mass of the trolley has been reduced by removal of the cylinders. From the wave-traces the accelerations are carefully calculated as in Exp. iii. If the

masses of the trolley in the two cases are as 2 : 1, then the corresponding *accelerations* are found to be in the proportion of 1 : 2. This result proves that if the force causing motion remain constant, the *acceleration is inversely proportional to the mass moved*[1].

EXAMPLES XII (SPEED, VELOCITY, AND ACCELERATION).

[N.B. In working out examples remember that if a body starts from rest, $u=0$, but if a moving body is brought to rest, $v=0$.]

1. Express a speed of 30 miles per hour in feet per second.

2. Express a speed of 88 feet per second in miles per hour.

3. A train starts from Doncaster at 10.40 A.M. and reaches King's Cross (156 miles distant) at 1.40 P.M. Calculate the average speed in (*a*) miles per hour, (*b*) feet per second.

4. A train starting from King's Cross at 2.20 P.M. and travelling at an average speed of 47 m. 1120 yds. per hr. reaches Edinburgh at 10.35 P.M. Find the distance travelled.

5. Express a speed of 16 metres per second in miles per hour.

6. A patch on a bicycle tyre makes 140 revolutions per minute. If the external diameter of the tyre is 28 in., calculate the speed in miles per hour.

7. A body starting from rest and moving with uniform acceleration traverses 72 ft. in 6 secs. What is (*a*) its acceleration, (*b*) its speed at the end of the 6 seconds?

8. A body starts from rest with a uniform acceleration of 8 ft. per sec. per sec. In what time will it acquire a speed of 24 ft. per sec.?

9. A body starts with a speed of 10 ft. per sec. and moves with an acceleration of 5 ft. per sec. per sec. What will its speed be (*a*) when it has traversed 30 ft., (*b*) after 5 seconds?

10. A particle moving in a right line with uniform acceleration has at a given instant a speed of 5 ft. per sec. and 10 secs. afterwards it has a speed of 35 ft. per sec. What is the magnitude of the acceleration? Over what distance does the particle move in the interval? O. J.

[1] In these trolley experiments, the mass moved includes the masses of the weights and pan, but these latter being small in comparison with the mass of the trolley may be neglected.

11. A stone is thrown vertically upwards with a velocity which will just carry it to a height of 100 ft. Calculate the velocity of projection and the time taken to reach the highest point.

12. A ball is thrown vertically upwards with a velocity of 80 ft. per sec After what interval will its velocity be 40 ft. per sec.?

13. A stone is dropped from a captive balloon and reaches the ground in 10 secs. How high is the balloon? If the balloon were moving upwards with a velocity of 32 ft. per sec. when the stone was dropped, what time would elapse before the stone began to fall?

14. A body falls from rest. Find the space described by it in the 4th second.

15. A stone is thrown vertically upwards with a velocity of 64 ft. per sec 2 seconds afterwards another is thrown up from the same place with the same velocity. When and where will they meet?

CHAPTER XIII.

COMPOSITION AND RESOLUTION OF FORCES.

76. Parallelogram of Velocities.

Exp. Put a marble at O in a glass tube OA (Fig. 94) and place the tube on a drawing board. Raise one corner of the board in such a way that the tube rolls across the board and at the same time the marble rolls down the tube. Let the tube and marble each move with uniform velocity. At the end of

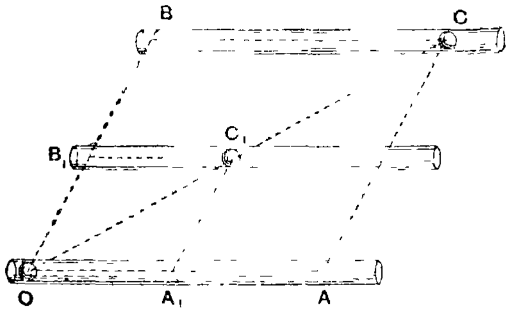

Fig. 94.

the first second let the tube roll a distance OB_1 and the marble roll a distance B_1C_1 down the tube. By the end of the second second the tube will be in the position BC where $OB = 2OB_1$, and the marble will have rolled to C, where $BC = 2B_1C_1$. It can be proved by geometry that O, C_1 and C are in the same straight line. The position of the ball at the end of the first and second

second could also be found by considering the displacements to take place separately, *i.e.* let the ball move along the tube for a second and come to rest at A_1, and then let the tube roll for one second, thus carrying the ball from A_1 to C_1, the velocities being the same as before. Similarly, let the ball move for two seconds from O to A, and the tube afterwards roll for two seconds from A to C. It is evident then that $OB = AC$ and $OA = BC$ and therefore that $OACB$ is a parallelogram, and that the actual path of the ball, when both displacements take place at the same time, is the diagonal OC. It is therefore clear that the resultant velocity of the two separate velocities superposed on the ball is represented by the diagonal of the parallelogram, both as regards magnitude and direction of the velocity.

Parallelogram of Velocities. *If a particle is given, at the same time, two velocities represented by two adjacent sides of a parallelogram, these velocities are equivalent to a single velocity, represented by the diagonal of the parallelogram passing through their point of intersection.*

Exercise. On squared cm. paper draw to scale a diagram showing the position of the ball at the end of 1, 3 and 4 secs., when the tube rolls in a direction at right angles to that of the ball, the velocity of the ball being 2 cm. per sec. and that of the tube 3 cm. per sec.; and show by accurate drawing that the path of the ball is a straight line.

77. Parallelogram of Forces.

We may apply the parallelogram of velocities to find the resultant of two *accelerations* simultaneously given to a particle, for acceleration is the change of velocity per second; and similarly we may extend the application further to find the resultant of two *forces* acting on a particle at the same time, since forces are proportional to the accelerations they produce in the same mass [§ 75 (3)].

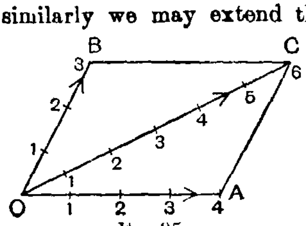

Fig. 95.

Let OA (Fig. 95) represent a force of 4 pounds acting in the

direction OA, and OB represent a force of 3 pounds acting in the direction OB.

Complete the parallelogram $OACB$, then the diagonal OC on the same scale represents in magnitude and direction (O to C) the resultant force, *i.e.* the single force which will replace the forces represented by OA and OB, and produce the same effect. Here $OC = 6$ units, and therefore the resultant force $= 6$ pounds acting in the direction O to C.

Parallelogram of Forces. If two forces acting on a particle be represented in magnitude and direction by two adjacent sides of a parallelogram, the resultant of the forces is represented by the diagonal of the parallelogram which passes through their point of intersection.

Equilibrium. If a number of forces acting on a particle produce rest, *i.e.* the particle does not move, the forces are said to be in *equilibrium*.

For instance the hub of a bicycle wheel at rest is kept in position by many forces acting along the spokes of the wheel and through the axle.

In Fig. 95, it is evident that if a force of 6 pounds in the direction C to O were substituted for the resultant OC, the three forces OA, OB and CO would be in equilibrium, the force CO being equal *but opposite* to the force OC.

78. Experiment to illustrate the parallelogram of Forces.

Attach thread to three sets of weights M, N and L (Fig. 96), and having knotted the threads together at O hang them over two pulleys G and F (as shown in the diagram) attached to a drawing-board. Pin a piece of paper to the board. Move O from its position of equilibrium and notice whether the threads again adjust themselves at the same angles. Carefully mark the directions of the forces P, Q, and R by two pencil marks on the paper along each of the threads. Note the magnitude of the

forces P, Q and R (*i.e.* record the weights L, M and N). Remove the paper and, taking suitable units, complete the parallelogram $OACB$ as illustrated in Fig. 95, § 77. The diagonal OC, which represents in magnitude and direction the resultant of forces

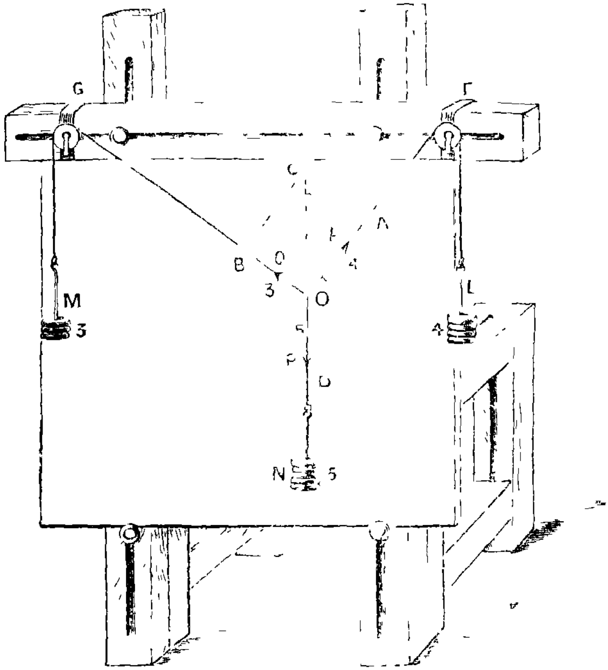

Fig. 96

represented by OA and OB, should be equal to R (in this case 5 units) in the line DO produced and opposite to R in direction.

[N.B. It is not necessary to take the same weights as are indicated in Fig. 96.]

Exercises.

(1) Find by a graphic method the resultant of two forces of 4 and 7 pounds weight acting from a point at an angle of 60° to each other (cf. Fig. 95).

(2) Find by a graphic method the resultant of forces 5 and 12 pounds weight acting on a particle at right angles to each other. Confirm your results by calculation.

(3) Two forces of 6 and 11 grams acting at a point have a resultant of 14 grams. Find by a graphic method the directions of the forces (cf Fig 95).

79. Resolution of Forces

To *resolve* a force in two directions is to find the two forces acting in the given directions which together have the same effect as the original force.

For instance, if we wish to resolve the force R represented by OC (Fig. 97) in the two directions OA and OB, we complete the parallelogram by drawing CA and CB parallel to OB and OA respectively. Then the components P and Q, which are together equivalent to R by the Parallelogram of Forces, are represented by OA and OB and they are called the resolved parts of R in the directions OA and OB

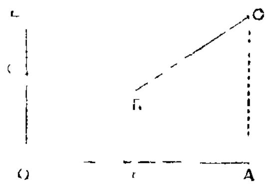

Fig 97

Exercises.

(1) OC in Fig 97 represents a pull of 36 tons in a wire rope; find the horizontal and vertical components, i.e. resolve OC horizontally and vertically (Use a mm. ruler.)

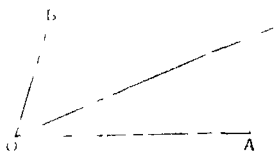

Fig 98.

(2) OC in Fig. 98 represents a force of 39 pounds; resolve this force in the directions OA and OB and measure the angles which the components make with the original force.

(3) Resolve both horizontally and vertically a force of 24 pounds acting upwards at an angle of 30° with the vertical

(4) Resolve a force of 20 grams into two components acting at angles of 30° and 45° with the direction of the 20 gram force. (Use a graphic method.)

80. Triangle of Forces.

Exp. i. Knot three strings together at O and tie the ends to three spring balances L, M, N (Fig. 99). Put the ring of N over a nail driven into a

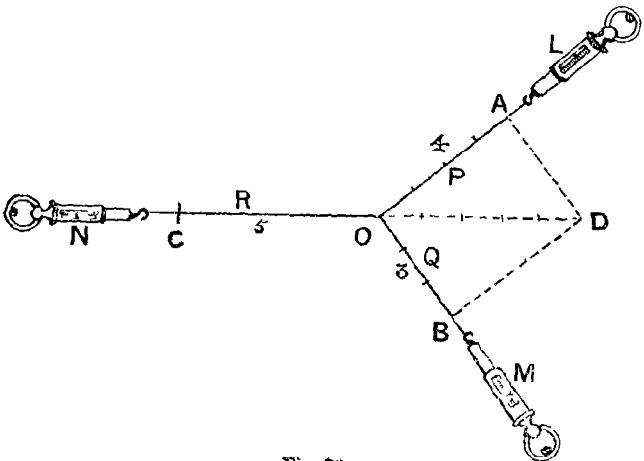

Fig. 99.

drawing board and, taking the rings of L and M in the two hands and pulling the strings tight, adjust the positions of L and M until the three forces acting at O are 4, 3 and 5 respectively. Fix L and M by driving nails through the rings. The tensions in the strings are *in equilibrium*. Note that the angles between the strings are equal to those in Fig. 96, § 78. Taking OA and OB equal to 4 units and 3 units respectively, as in § 78, *complete* the parallelogram $OADB$. OD should be equal to 5 units and should also be in the same straight line with CO. AD is equal to OB and represents 3 units.

Consider how the triangle OAD (Fig. 100) represents the three forces in equilibrium.

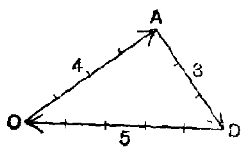

Fig. 100.

The side OA represents in magnitude and direction a force of 4 along OA.
The side AD represents in magnitude and direction a force of 3 along OB.
The side DO represents in magnitude and direction a force of 5 along OC.

Notice also that the sides OA, AD and DO are *taken in order*.

Exp. ii. Next move L and M to new positions. Record the new tensions indicated by the spring balances and draw a triangle having its sides parallel to the directions of the three forces. If the pull recorded by one of the balances is (say) 80 grams, make the corresponding side of the triangle 80 mm. in length. Then the lengths (in mm.) of the remaining two sides will correspond to the tensions (in gms.) of the other two strings. Note too the direction of the tensions in the strings is the same as the direction of the sides of the triangle taken in order.

Hence we obtain the following general statement :—

Triangle of Forces. If three forces acting on a particle are in equilibrium, they can be represented in magnitude and direction by the sides of a triangle taken in order.

The converse is also true. If three forces acting at a point can be represented in magnitude and direction by the sides of a triangle taken in order, they are in equilibrium.

81. Practical Problems.

1. Knot three strings together at O (Fig. 101) and hang a weight W

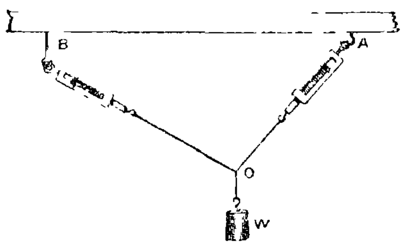

Fig 101.

by two spring balances A and B. *Find the tensions in the strings OA and OB.* Draw a vertical line $CD = W$ units and construct a triangle CDE, having DE parallel to OA and EC parallel to OB. On the same scale measure DE and EC; the tensions measured by the spring balances at A and B should be in proportion to the lengths of DE and EC.

2. **Funicular Polygon.** By means of a string tied to two nails A and B (Fig. 102) suspend three weights W_1, W_2, W_3. Knowing the weight of W_1, *find the weights of W_2 and W_3 and the tensions in the strings.* As in the last problem draw a vertical line X_1X_4 and measure off $X_1X_2 = W_1$ units, complete the triangle X_2X_1O by drawing X_1O parallel to A_1A and OX_2 parallel to A_1A_2, then the three sides of the triangle X_2X_1O represent the three forces in the strings

which keep the point A_1 in equilibrium. Continue[1] by drawing OX_3 parallel to A_2A_3; then in the triangle X_3X_2O, X_3X_2 represents W_2 and X_2O represents T_2 and OX_3 represents T_3; and so on.

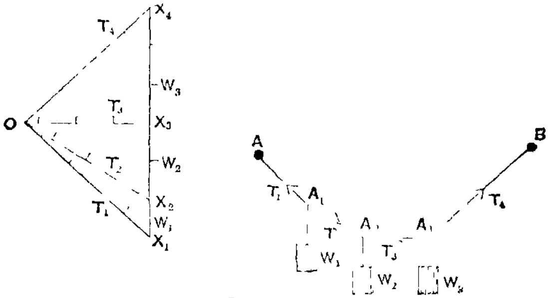

Fig. 102.

In this particular case, given that W_1 weighs 1 pound, it is found by measurement from the diagram, taking $X_1X_2 = 1$ unit of length, that:—

$OX_1 = 4\cdot4$ units, $OX_2 = 3\cdot6$ units, $X_2X_3 = 2$ units, &c.,

and therefore, correspondingly,

$T_1 = 4\cdot4$ pounds, $T_2 = 3\cdot6$ pounds, $W_2 = 2$ pounds, &c.

Examples XIII (Composition and Resolution of Forces).

1. Represent graphically forces of 15 lb. wt. and 25 lb. wt. acting at an angle of 120°, and find their resultant by measurement.

2. A horse exerts a pull of 150 lb. wt. on a railway truck at 30° to the direction of the rails. Find the force pulling the truck forwards and that tending to pull it off the rails.

3. Forces of 5 lb. and 12 lb. wt. act at an angle of 90° to each other at a given point. Find the direction and magnitude of the resultant.

4. If the resultant of two forces acting at right angles is $2\frac{1}{2}$ lb. wt. and one of the forces is $1\frac{1}{2}$ lb. wt., what is the other force?

5. Forces of 2, 4, 6, 8 lb. wt. act at a point at right angles to one another. Find the direction and magnitude of their resultant.

[1] [N.B. When the equilibrium of the point A_2 is considered, the directions of tensions T_2 and T_3 are *away from* A_2.]

6. A force of 8 lb. wt. acting vertically upwards is resolved into two forces, one being horizontal and equal to 6 lb. wt. What is the magnitude and direction of the other component?

7. The resultant of two forces, one of 8 lb. wt., the other 17 lb. wt., is a force at right angles to the smaller. Determine the magnitude of the resultant. Mark on a diagram the directions of all the forces. O. J.

8. A weight of 20 lb. is suspended by a string and is pulled horizontally by a force of 15 lb. wt. Find (*a*) the angle the string makes with the vertical, (*b*) the tension in the string.

9. A weight of 7 lb. is hanging from a string which passes over a pulley and is then fixed to a wall. If the string from wall to pulley is horizontal what is the magnitude and direction of the thrust on the pulley?

10. A weight of 10 lb. is supported by two strings which make angles of 30° and 60° with the vertical. Find the tensions in the strings.

11. A rod AB 1 ft. long is hinged at A and kept in a horizontal position by a string BC fixed to a point C 1 ft. above A. A weight of 10 lb. is hung at B. Find the tension in the string neglecting the weight of the rod.

CHAPTER XIV

MOMENTS, PARALLEL FORCES, COUPLES, CENTRE OF GRAVITY.

*82. Experiment on Moments.

The uniform rod AB is hung by a hook at its middle point C (Fig. 103). When no weights are suspended on the rod, it

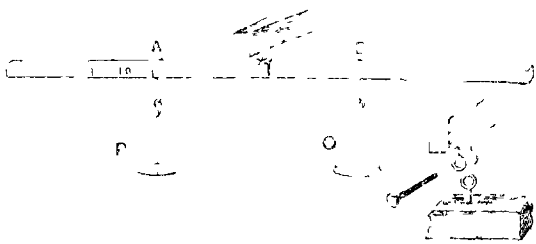

Fig. 103.

should hang horizontally. Sliding rings A and B, with weight pans P and Q attached, may be placed at any position on the rod.

(1) It is evident that if only one pan Q is attached, the rod tends to turn in a "**clockwise**" direction; but if the pan P alone is attached the rod tends to turn in a "**counter-clockwise**" direction

(2) If the rings and pans are of *equal weight* and are hung at equal distances from C, we have all the essential features of a **balance** (see § 27).

(3) We know that on a see-saw a light boy can balance a heavy boy if the lighter boy can get considerably further away from the turning centre than the heavier one. So also a greater weight hung at B can be balanced by a smaller weight hung at A, but when a balance is obtained it will be found that $AC > CB$.

(4) *Keeping the point A fixed*, 8 inches from C, place 30 grams in pan P[1]. Place Q 6 inches from C and add weights until a balance is obtained. 40 grams must be added to the pan Q[1]. Repeat the experiment placing first 60 and then 90 grams in P, but *keep the weights in Q the same* (40 *grams*), moving the pan until a balance is obtained and record your results as follows:

1.	2	3	4	5	6
Weight at A[1]	"Arm" CA	Force × arm	Weight at B	"Arm" CB	Force × arm
30 grams	8 inches	30 × 8 = 240	40 grams	6 inches	40 × 6 = 240
60 ,,	8 ,,	60 × 8 = 480	40 ,,	12 ,,	40 × 12 = 480
90 ,,	8 ,,	90 × 8 = 720	40 ,,	18 ,,	40 × 18 = 720

(5) Repeat (4) keeping the arm CA *constant at* 12 *inches* and keeping the *weight at B constant at* 60 *grams*. Begin with a weight of 20 grams in P[1] and increase it by 20 grams each time.

N.B. The bar must be *horizontal* each time the readings are taken.

The results of (4) and (5) show that the tendency of a force to turn the bar about the point C depends on

(*a*) the *magnitude of the force*,

(*b*) the perpendicular distance of its line of action from the point C, *i.e.* on the *length of the "arm"*.

[1] Include the weight of the pan. To facilitate work the pan and ring should be made of light material (*e.g.* aluminium) and should weigh exactly 10 grams. The weights given in columns (1) and (4) include the weights of the pans.

When the product "force × arm" is the same on both sides of C, a balance is obtained. This product is called the moment of the force about the point.

The turning moment or moment of a force about a point is the product of the force and the perpendicular let fall from the point on to the line of action of the force.

83. Principle of Moments.

If any number of forces acting in a plane are in equilibrium the algebraic sum of their moments about any point in their plane is equal to zero.

To illustrate this statement let us make a few experiments.

***Exp. i** Using the same rod as in § 82, suspend it by its middle point C. Let W be its weight. Since the bar balances at C, its weight W must act through C. Therefore the moment of W about $C=0$. Suspend several weights on the bar adjusting their position (Fig. 104) so that the bar hangs

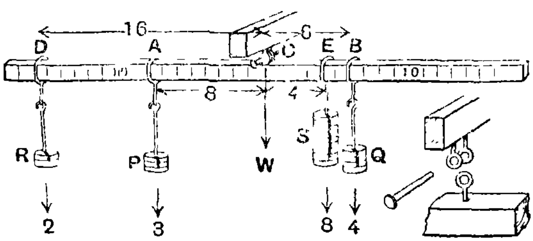

Fig. 104.

horizontally. Take moments about C, "clockwise" direction being +, "counter-clockwise" −. Then

$$Q \times CB + S \times CE + W \times 0 - P \times CA - R \times CD = 0,$$

here $$4 \times 6 + 8 \times 4 + 0 - 3 \times 8 - 2 \times 16 = 0.$$

***Exp. ii.** Hang the rod on the hook as before at C. Place weights in pan P; tie a string, to which a dynamometer is attached, at B (Fig. 105). Holding the dynamometer in various positions (BQ, BQ_1, &c.) in the same vertical plane as the rod and so that AB is horizontal, take readings of the

tension shown by the dynamometer and the corresponding perpendicular distance from C to the direction of the string from B. Then taking moments about C, show that

$$Q \times Cq + W \times 0 - P \times AC = 0.$$

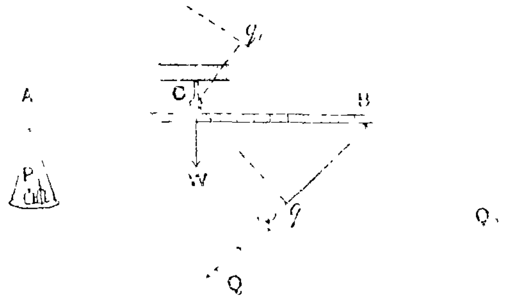

Fig. 105.

*Exp. III. Using the same rod, attach two strings at A and B and, having tied two light spring balances to the strings, pull them lightly so that the bar still remains horizontal (Fig. 106). Support the balances

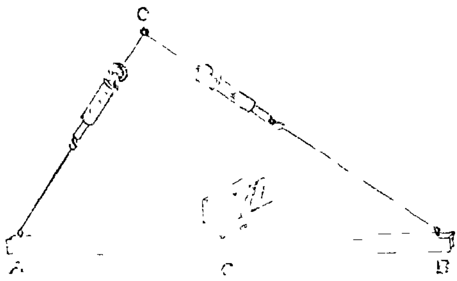

Fig. 106

slightly to relieve the string of the tension due to their weight and take readings of the pulls recorded. Take moments about C. Measure the perpendicular distances from C to AO and BO. It will be found that:

Tension in $AO \times$ perpendicular from C to AO

$= $ Tension in $BO \times$ perpendicular from C to BO.

(The moment of W (the weight of the bar) about $C = 0$.)

*Exp. iv. Set up the apparatus as in § 78, Fig 96 Take any point C (Fig. 107) and let fall perpendiculars Cp, Cq, Cr on to the line of action of

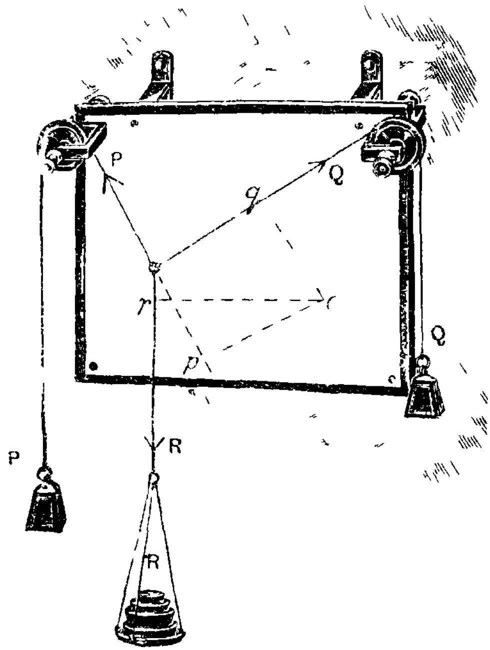

Fig. 107.

the three forces P, Q and R which are in equilibrium Carefully mark the directions and magnitude of the forces on the drawing board. Take moments about C and show by calculation that·

$$P \times Cp + Q \times Cq - R \times Cr = 0.$$

84. Graphic Representation of a Moment.

Let a force F be represented in magnitude and direction by the line AB (Fig 108) Let C be a point about which the moment is taken. Let fall a perpendicular CD from C on to the

direction of the force. Join CA and CB. Then the moment of F about $C = F \times CD$

$$= AB \times CD = 2\triangle ABC.$$

Therefore the **moment** of a force may be **represented graphically** by **an area**.

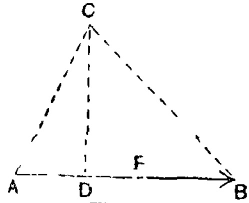

Fig. 108.

N.B. CD is called the Arm. In this case the moment is "counter-clockwise" in direction.

***Exercises.** Show "graphically" by use of squared paper that *the algebraic sum of the moments of two forces about a point in their plane is equal to the moment of their resultant about that point.*

In Figs. 109 *a* and 109 *b*, R is the resultant of P and Q (§ 77) and H is the point about which moments are taken. HBC is drawn parallel to OA. OB is then taken to represent Q and OA to represent P on the same scale. Complete the parallelogram $OACB$, then the diagonal OC represents R on the same scale. Join H to O and to A. Show that:

$$2\triangle HOC = 2\triangle HOB + 2\triangle HOA \text{ (Fig. 109 } a\text{)},$$
$$2\triangle HOC = 2\triangle HOA - 2\triangle HOB \text{ (Fig. 109 } b\text{)}.$$

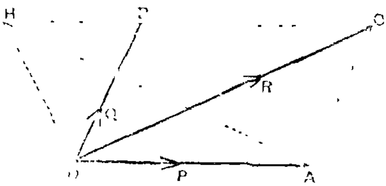

Fig. 109 *a*.

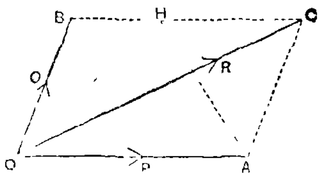

Fig. 109 *b*.

85. Parallel Forces. When parallel forces act in the same direction they are said to be **like**, if in opposite directions they are said to be **unlike**.

If in § 82, Fig. 103, we had replaced the hook at C supporting the rod by a spring balance, it is evident that we could have determined at once the magnitude of the force which kept in equilibrium the parallel forces P and Q acting downwards. It is therefore clear that the **resultant** of the two parallel forces P and Q acts at C and $= P + Q$. By the principle of moments we also can find the **position** of C, for $P \times CA = Q \times CB$.

***Exp. To find the position and magnitude of the resultant of two parallel forces.**

(*a*) **Like parallel forces.** Suspend the rod AB horizontally and at equal distances from its ends by two circular[1] balances D and E (Fig. 110).

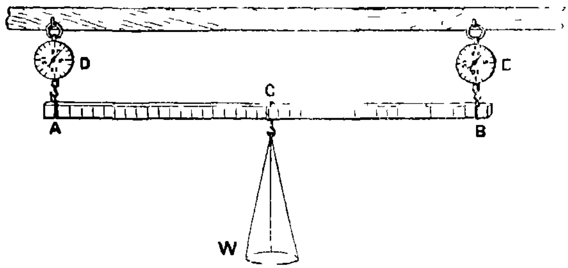

Fig. 110.

Note the readings, which should be the same and equal to w where $2w$ = wt. of rod. Put a suitable weight in the pan $=W$. Take readings P and Q of the circular balances (subtracting w from each reading) and record the distances AC, CB. Move C to various positions on the bar and repeat. As before we find that $W=P+Q$ but is opposite in direction, therefore **resultant** $R=P+Q$ in the **same direction** and the position of the resultant is such that

$$P \times CA = Q \times CB.$$

(*b*) **Unlike parallel forces.** Using the same reading as in (*a*), P and W are two unlike parallel forces; the force Q balances the resultant of P and W and acts upwards, therefore the **resultant** R of P and W acts downwards and $= W-P$. The position Q is determined by the *principle of moments*, by taking moments about B, then, for equilibrium,

$$\text{moment of } P \text{ about } B = \text{moment of } W \text{ about } B,$$

$$\therefore\ P \times BA = W \times BC.$$

The *resultant* both in *like* and in *unlike* parallel forces is the *algebraic sum of the forces*. The *position* of the resultant is obtained by finding a point about which the moments of the two forces are equal but opposite.

86. Couples. A thoughtful student may ask "what becomes of the resultant when we are dealing with *two equal*

[1] Ordinary dynamometers will serve the purpose, but it is more difficult to keep the rod horizontal.

but unlike parallel forces?" as for instance P, P in Fig. 111. Take moments about any point O in the plane of the forces. Draw through O, OAB perpendicular to the direction of the forces. Then the algebraic sum of the moments of P and P about O

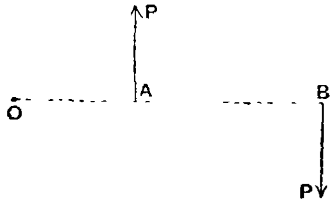

Fig. 111.

$$= P \times OB - P \times OA$$
$$= P(OB - OA) = P \times AB.$$

$P \times AB$ is called the moment of the couple PP.

A couple consists of two equal but unlike parallel forces and has no single force as a resultant.

***Exp. To show that a couple and a single force acting on a body cannot keep it in equilibrium,** *i.e.* that a couple cannot be balanced by a single force.

A metre ruler AB is placed on a board: *no pivot* must be placed at C, its middle point, but we can imagine that the ruler is arranged to rotate to a limited extent about C (Fig. 112). Drive four nails, d, e, f and g into the board at equal distances from C and arrange them to prevent the ruler from rotating about C more than 2 or 3 inches. Attach two spring balances at equal distances from C and secure them by nails at h and j in such a way that the tensions are the same in both and in parallel but opposite directions. The rod is now pulled as shown in Fig. 112 against the nails e and f by a couple whose moment $= P \times AB$. The nails e and f prevent the ruler from rotating about C to a greater extent in a counter-clockwise direction.

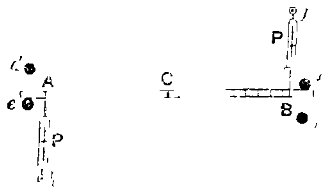

Fig. 112.

Attach a third spring balance and, by pulling in the same plane as the board, try if it is possible to move the ruler away from *both* the nails e and f at the same time.

If this attempt fails, use a fourth spring balance and find whether the second pair can be used together to counterbalance the original pair of forces.

If a balance is obtained, find whether the second pair constitute a couple and under what conditions.

The results show that to produce equilibrium with a couple, a second couple must be applied.

87. Centre of Gravity.

Consider a body such as a stone. We can imagine that it is made up of a great number of particles. The *weight* of the stone is equal to the sum of the weights of the particles. As each particle is attracted to the earth's centre those weights constitute a system of practically parallel forces. We understand how to find the resultant of two of these parallel forces and having found this single force we can, with a third parallel force, find a resultant which shall equal the three forces and so on with a fourth; finally a single force will be found which is the resultant of the weights of all the particles. *This resultant, equal to the weight of the body, passes through a point which is fixed in the body however it is placed.* This fixed point is called the **centre of gravity** of the body

***Exp. To find the centre of gravity of a thin uniform board.**

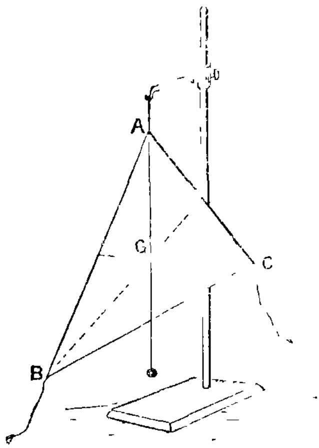

Fig. 113.

Several pieces of irregularly shaped mill-board are supplied. Three or four pieces of thread are attached at different points in each board. Hang one of the mill-boards to a hook from which a plumb-line is suspended. Allow the mill-board and plumb-line to come to rest. The centre of gravity (C.G.) must lie directly under the hook in the line of the plumb-line. Mark this line on the mill-board. Repeat the process, hanging the board up by another thread (see Fig. 113). A second line is thus found which also contains the C.G. Where the two lines cross must be the position of the C.G. Suspend the board by a third thread and note whether the plumb line passes through the point of intersection of the two lines.

In Fig. 113 the method of finding the C. G. of a triangular plate is shown. The C. G. is found to be $\frac{1}{3}$ up the median line of the triangle.

88. To find by geometry the centre of gravity of thin plates cut in the shape of (*a*) a circle, (*b*) a square, (*c*) a parallelogram, (*d*) a triangle.

(*a*) and (*b*) In the case of symmetrical figures, *e.g.* a **circle** and a **square**, we can draw lines through the *centre* dividing the figure into two identical halves. It is therefore evident that for every particle on one side of this line there is a corresponding particle on the other side, of equal weight and at the same distance from the centre. Therefore the C. G. must lie at the *centre* of a circle or of a square.

(*c*) This process of "*symmetry*" may be applied to finding the C. G. of a **parallelogram.** A uniform rod "by symmetry" has its C. G. at its centre. Divide the parallelogram *ABCD* into rods, *e.g.* *PRQ*, parallel to *AB* (Fig. 114). The C. G. lies along the median line *EF*. Again, dividing the parallelogram into rods parallel to *AD*, the C. G. lies on the median line *KH*. Therefore the C. G. lies at the point of intersection of *EF* and *KH*, *viz.* at *G*.

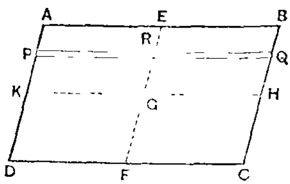

Fig. 114.

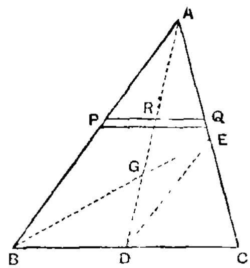

Fig. 115.

(*d*) Confirm the result of Exp., § 87, by the method of "symmetry" applied to find the C. G. of the **triangle** *ABC* (Fig. 115). Divide the triangle up into rods parallel to the side *BC*. The C. G. lies on the *median* line *AD*. Similarly dividing the

triangle up into rods parallel to the side CA, the C.G. lies on the median line BE. Therefore the C.G. of triangle ABC lies at G, the point of intersection of the median lines.

N.B. By geometry, since $2DE = AB$, and since triangles ABG and DEG are similar,

$$\frac{AG}{GD} = \frac{AB}{DE} = \frac{2DE}{DE} = \frac{2}{1}.$$

Therefore G is $\frac{1}{3}$ up the median line.

89. Given the centres of gravity of each of two parts of a body to find the C.G. of the whole.

Let G_1 and G_2 (Fig. 116) be the centres of gravity of the right and left hand parts respectively of a body divided by the vertical line: join G_1G_2. The C.G. must lie on G_1G_2 and will be at such a point (G) that, by § 85,

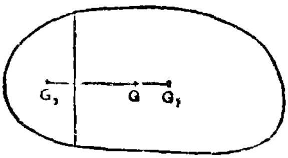

Fig. 116.

wt. of right-hand part × GG_1
= wt. of left-hand part × GG_2

*Exp. i. Apply this method to find the C.G. of two *circular* mill-boards of *uniform* thickness glued together (Fig. 117). Find the C.G. (*a*) by *calculation*, (*b*) by *experiment* (suspension by threads).

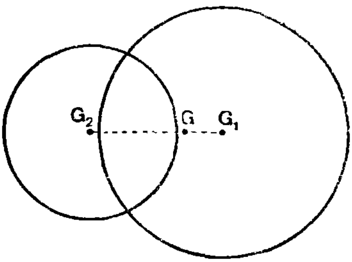

Fig. 117.

*Exp. ii. *Find* (*a*) by calculation, (*b*) by experiment (suspension) the C.G. of a piece of uniform wire *ABC bent at right angles* at B and of dimensions—$AB = 8$ inches, $BC = 6$ inches.

N.B. The C.G. does not lie in the wire itself

Examples XIV a (Moments).

1. A 5 lb. wt. is suspended from one end of a weightless rod 8 ft. long. If the point of support is $3\frac{1}{2}$ ft. from the 5 lb. wt. what weight must be hung from the other end to obtain a balance?

2. At what point must a bar 10 ft. long be supported so that a weight of 9 lb. at one end will balance a 6 lb. wt. at the other?

3. A weightless rod has a 5 lb. wt. suspended at one end and a 10 lb. wt. at the other. If a balance is obtained when the point of support is 4 in. from the 10 lb. wt. what is the length of the rod?

4. Two weights are hung from the ends of a weightless rod: the pressure on the supporting hook is 9 lb., one of the weights is 6 lb. and the length of the shorter arm is 5 inches. What is the length of the rod?

5. 5 gm. and 20 gm. are hung at the ends of a metre ruler. Neglecting the weight of the ruler, what weight must be hung 25 cm. from the 5 gm. wt. to cause the ruler to balance at its mid-point?

6. A weightless rod 1 yd. long has weights 1 lb., 2 lb., 3 lb., 4 lb. hung at equal distances apart. Find the point at which the beam will balance.

7. A metre ruler has a 10 gm. wt. hung at the end and is found to rest horizontal when supported at a point 5 cm. from the centre. Find the weight of the ruler.

8. Two weights, 5 lb and 30 lb., are hung from the ends of a uniform rod 2 ft. long, which will then balance about a point 6 in. from the 30 lb. wt. Find the weight of the rod.

Examples XIV b (Parallel Forces).

1. Find the magnitude and the position of the resultant of the following *like* parallel forces:

(*a*) 5 lb. wt. and 7 lb. wt. at a distance of 12 inches.

(*b*) 50 gm. wt. and 20 gm. wt. at a distance of 14 cm.

2. Find the magnitude and the position of the resultant of the following *unlike* parallel forces:

(*a*) 2 lb. wt. and 11 lb. wt. at a distance of 6 inches.

(*b*) 10 gm. wt. and 20 gm. wt. at a distance of 10 cm

3. Two unlike parallel forces P and Q have a resultant of 6 lb. wt. acting 4 ft. from Q which is 14 lb. wt. Find the magnitude and position of P.

4. A beam 2 ft. long, weighing 14 lb., rests on a smooth peg at one end and is kept horizontal by a vertical cord attached 3 in. from the other end. Find (*a*) the tension in the cord, (*b*) the pressure on the peg.

5. If a load of 100 lb. is hung from a weightless bar 3 ft. long resting on a smooth peg at each end, what is the pressure on the pegs when the load is (*a*) at the centre of the bar, (*b*) 6 in. from one peg?

6. A bar AB, 6 ft long, weighing 11 lb., rests on two pegs, one at A, the other 6 in. from B. Find the pressure on each peg.

7. A man and a boy are carrying a load of 90 lb. slung from a pole, 6 ft long, weighing 10 lb. Where should the load be placed so that the man may support ⅔ of the total weight?

8. A uniform rod 12 ft. long, weighing 8 lb., rests on supports 8 ft. apart. If the pressure on one support is 5 lb. wt., find the distance of the supports from the ends of the rod.

9. A bar, AB, 4 ft. long, weighing 10 lb., is hung by two vertical wires attached at the ends. If the wire at B can only support a load of 10 lb., how near to B may a weight of 20 lb. be hung safely?

EXAMPLES XIV C (CENTRE OF GRAVITY).

1. Four weights, 2 lb., 4 lb., 6 lb. and 8 lb., are placed 1 ft. apart along a uniform rod 3 ft. long. If the rod weighs 10 lb. find the position of the C. G.

2. A metre ruler weighing 95 gm. rests on a table with part projecting over the edge. Find the length of this part if a 5 gm. wt. hung at the end just tilts the ruler.

3. A tapering rod 4 ft. long and weighing 6 lb. will balance at its mid-point when ½ lb. is hung on the end. At what point would it balance if the ½ lb. were removed?

4. A wire is bent to form a right angle ABC. AB is 12 in. long and BC 5 in. Find the position of the C. G.

5. A wire is bent into an isosceles triangle the equal sides being 13 in. long and the base 10 in. Find the distance of the C.G. from the mid-point of the base.

6. If three equal weights be placed at the corners of a triangular lamina what is the distance between the centres of gravity of the weights and lamina. Prove your statement.

7. The diagonals of a square (side 6 in.) are drawn and one of the triangles so formed is cut out. Find the position of the C. G. of the remainder.

8. A lamina has the form of a rectangle of length 6 in. and breadth 4 in. with an isosceles triangle [illegible] in. high on one of the 6 in. sides. How far from the C. G. of the rectangle will the C. G. of the whole lamina be?

9. The mid-points of two adjacent sides of a square are joined by a line and the triangle so formed is cut off. Find the position of the C. G. of the remainder.

10. A circular hole of 4 in. radius is cut out of a circular disc of cardboard of 1 ft. radius. If the centre of the hole is 3 in. from the centre of the disc, where does the C. G. of the remainder lie?

CHAPTER XV.

PRINCIPLE OF WORK, SIMPLE MACHINES.

90. A **machine** is a contrivance by means of which a force called the **effort** exerted at one point is arranged to overcome a resisting force called the **resistance** or *load* acting at a different point. The ratio between *resistance* and *effort* is called the **mechanical advantage** of the machine,

$$\frac{\text{resistance}}{\text{effort}}, \ i.e.\ \frac{\text{load}}{\text{force}} = \text{mechanical advantage.}$$

The simplest machine is the lever. We know that one form of *lever* is a bar used for prising up heavy weights. Thus if a force of 5 pounds exerted at one end of a crowbar used as lever raises a block of stone weighing 20 pounds,

the *mechanical advantage* of the lever

$$= \frac{\textit{resistance}\text{ of 20 pounds}}{\textit{effort}\text{ of 5 pounds}} = 4.$$

Work.

A force does work when it moves its point of application.

A force causing no movement does no work.

The amount of **work** done is measured by the **product** of the **force** (F) and the **distance** (s), through which its point of application is moved *in the direction of the force.*

$$\text{Work} = \text{force} \times \text{distance} = Fs.$$

Imagine that a smooth stone is pulled by a force F up an ice

slope AB (Fig. 118), then the **work done by** $F = F \times AB$
Allow the stone to slide down the length of the ice slope, the pull of the earth on the stone, *i.e.* its weight W, is doing work on the stone. The weight acts *vertically* downwards and therefore

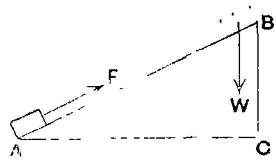

Fig. 118.

the **work** done on the stone by gravity

$$= W \times \text{vertical fall} = W \times BC.$$

N.B. Work done by F in moving stone from A to $B = F \times AB$
" " W " " B to $A = W \times BC$.

When we move a lever we do work **on** the machine; the lever raises a stone (say), work then being done **by** the machine. In a perfect machine all the work done *on* the machine should appear as work done *by* the machine.

Thus if we apply a force F to the handle of the lever and F acts through a distance s, the work done *on* the machine $= Fs$. If the load of weight W is raised vertically a distance d, then the work done *by* the machine $= Wd$.

The Principle of Work states that in a perfect (*i.e.* a frictionless machine)

$$Fs = Wd,$$

or that

the *work done* **on** the machine = the *work done* **by it.**

Efficiency.

The ratio $\dfrac{\text{work done } by \text{ machine}}{\text{work done } on \text{ machine}}$ is called the **efficiency of** the machine, hence in a *perfect* machine $\dfrac{Wd}{Fs} = 1$.

But practically, **friction** interferes with efficiency and reduces the work done *by* the machine, hence in practice the **efficiency** is always <1. In other words, part of the work done *on* the machine is used in overcoming friction.

To find the mechanical advantage $= \dfrac{\text{resistance}}{\text{effort}}$.

Make use of the relationship established by the Principle of Work, viz. $Fs = Wd$, which in words may be stated as follows:
Effort × distance through which the Effort acts
= Resistance × distance through which Resistance acts.

$$\therefore \text{ Mechanical advantage} = \frac{\text{Resistance}}{\text{Effort}}$$

$$= \frac{\text{Distance through which Effort acts}}{\text{Distance through which Resistance acts}}$$

$$= \frac{W}{F} = \frac{s}{d}.$$

The ratio $\frac{W}{F}$ or $\frac{s}{d}$ is sometimes called the **Velocity Ratio,** since it expresses the ratio between the distances through which the Effort and the Resistance move in unit of time.

91. Friction.

The force of friction always *opposes motion.*

Friction depends on the *nature of the surfaces* in contact.

We know, for instance, that it is easier to slide on smooth than on rough ice, that a wet leather cricket ball slips from the fingers more readily than a dry one, that an oiled bicycle is worked more easily than an unoiled one, and so on

***Exp. 1. To show that the friction is proportional to the pressure between the surfaces in contact.**

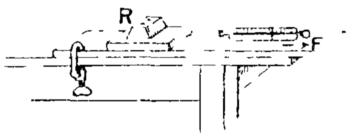

Fig. 119.

On a smooth level piece of wood or plate glass place a smooth block of wood of known weight to which is attached a spring balance (Fig. 119). Pull

the balance *slowly* in a horizontal direction and observe (*a*) the reading when the block begins to move, (*b*) the reading when the block is pulled along at a uniform speed. Notice that the (*a*) readings—*starting or limiting friction*—are greater than the (*b*) readings—*sliding friction*. Take the *mean* of several readings. Gradually increase the *pressure* (*R*) between the surfaces, *i.e.* the weight of block and load, and record your results as follows:

R = wt. of block + load	F = Force required to move the block	Ratio $\frac{F}{R}$
100 + 50 = 150	20 grams	$\frac{20}{150} = \cdot 133$
100 + 100 = 200	27 grams	$\frac{27}{200} = \cdot 135$
100 + 150 = 250	34 grams	$\frac{34}{250} = \cdot 136$
&c.	&c.	&c.

The ratio $\frac{\text{Friction}}{\text{Pressure}}$ is called the **Coefficient of friction.**

***Exp. ii** **To show that, keeping (*R*) the pressure between the surfaces constant, the Friction (*F*) does not vary with the area in contact** provided that the nature of the surfaces does not change.

Repeat the last experiment, using a rectangular block of hard wood with three unequal faces all made as smooth as possible. Try whether a different value of F is obtained by changing the surface of the block in contact with the board or plate. The load in each case must be kept constant.

92. The **mechanical powers** are—the **lever,** the **wheel** and **axle**, the **pulley,** the **inclined plane,** the **wedge** and the **screw.**

In our treatment of the *lever* and the *wheel and axle* we shall make use of the *principle of moments* and in dealing with the *pulley* we take it for granted that the *tension* in a string passing over a smooth surface is the *same throughout* its length. The *wedge* and the *screw* are *modifications* of the *inclined plane.*

The **Lever.**

In describing the *balance* (§ 27) and in our experiment on *moments* (§ 82) we were dealing with the essential feature of a *lever*: a rigid rod or bar turning about a point called the

fulcrum. There are three kinds of levers depending on the position of the fulcrum.

I. In the *First Order* the *fulcrum* is between the point where the *effort* (F) is applied and the point where the *resistance* is overcome or the *load* (W) lifted (Fig. 120).

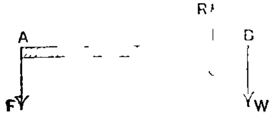

Fig. 120.

Let AB be a rigid bar moving about a fulcrum C. The *force* or *effort* (F) is applied at A and the load (W) is raised at B. Take moments about C. By the principle of moments, when there is equilibrium,

$$F \times CA = W \times CB,$$

$$\therefore \text{ the } \textit{mechanical advantage} = \frac{W}{F} = \frac{CA}{CB}.$$

Instances: the balance, the steelyard, a pair of scissors, a pair of pincers, crowbar raising stone at its end, etc.

II. In the *Second Order* the *load* (W) is between the *fulcrum* (C) and the point of application of the *effort* (F) (Fig. 121).

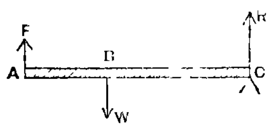

Fig. 121.

As before, taking moments about C, the *mechanical advantage*

$$= \frac{W}{F} = \frac{CA}{CB}.$$

Instances: a crowbar (its end resting on the ground), a wheelbarrow, nut-crackers, pump-handle (pivoted at end), an oar.

III. In the *Third Order* the *effort* (F) is applied between the *load* W and the *fulcrum* (C) (Fig. 122).

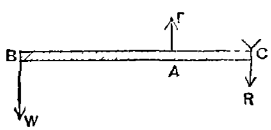

Fig. 122.

Again, taking moments about C, the *mechanical advantage*

$$= \frac{W}{F} = \frac{CA}{CB}.$$

Instances: the fore-arm, a pair of shears, a pair of short tongs.

N.B. Applying (i) the *Principle of Work* or (ii) the *Velocity Ratio*, we can

obtain the *mechanical advantage* by showing that the distances moved by *F* and *W* in small displacements are proportional to their distances from *C*

Practical Exercise. Revise the experiments in § 82 and (*a*) use *spring balances* in illustrating Orders II and III for *W* and *F* respectively, or (*b*) tie a string between the lever and pan, passing the string over a pulley (see Fig. 123).

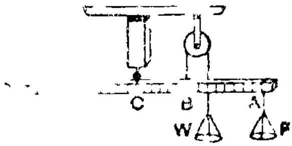

Fig 123

93. The Wheel and Axle consists of a large wheel

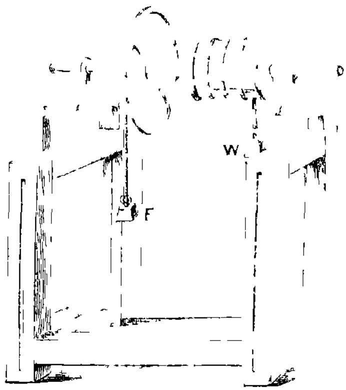

Fig. 124

(Fig. 124) to which (*F*) the effort is applied and a smaller wheel or barrel, also on the same axis, to which the load (*W*) to be raised is attached. Fig 125 shows a section

Taking *moments* about *C*, the axis,

$$F \times CA = W \times CB,$$

∴ the *mechanical advantage*

$$= \frac{W}{F} = \frac{CA}{CB}$$

$$= \frac{\text{Radius of Wheel } (R)}{\text{Radius of Axle } (r)}.$$

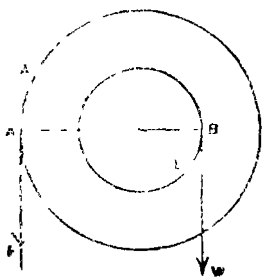

Fig 125

N.B. Applying (i) the *Principle of Work* or (ii) the *Velocity Ratio*, we obtain the *mechanical advantage* by imagining a small displacement AA' to take place, for in a small displacement, AA' may be regarded as in the same direction as F.

Then work done *on* the machine = work done *by* the machine.

$$\therefore\ F \times AA' = W \times BB'.$$

$$\therefore\ \frac{W}{F} = \frac{AA'}{BB'} \text{ and } \frac{AA'}{BB'} \text{ is proportional to } \frac{CA}{CB} = \frac{R}{r}.$$

94. The Pulley is a grooved wheel (Fig 126) which turns about an *axis* C passing through the centre. In Fig. 126, the axis is shown fitted into a *block* which is hung to a beam. Such a pulley is said to be *fixed* as distinguished from a *movable* pulley, which is free to change its position in space. A single fixed pulley, as shown in Fig. 126, merely changes the direction of the force F. Taking moments about C,

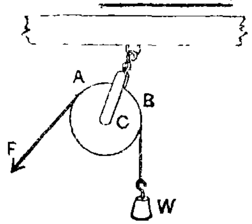

Fig. 126.

$$F \times CA = W \times CB \text{ but } CA = CB.$$

$$\therefore\ F = W \text{ and the mechanical advantage} = \frac{W}{F} = 1.$$

In practice, owing to friction, the tensions on both sides of the pulley are not equal, $F > W$ by the force required to overcome friction, if W is about to rise.

***Exp.** (*a*) In Fig. 127, a *movable* pulley is shown. If spring balances are inserted (1) between B and C and (2) between A and D, it will be seen that each registers a pull of $\frac{W+w}{2}$, where w = wt. of pulley,

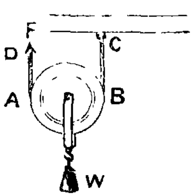

Fig. 127.

$$i.e.\ W + w = 2F,$$

or the Mechanical Advantage $= \frac{W+w}{F} = 2$.

(*b*) Prove that this result is correct by the *Principle of Work*. Raise W through a vertical height of x inches; A and B will each be raised by x inches, and therefore,

to keep the string tight, D must be raised $2x$ inches But work done *on* machine = work done *by* machine

$$\therefore F \times 2x = (W + w) x$$

$$\frac{W + w}{F} = 2$$

Exp "The 1st system" of Pulleys, often described but not generally useful, is shown in Fig 128 Strings round three movable pulleys W_1, W_2

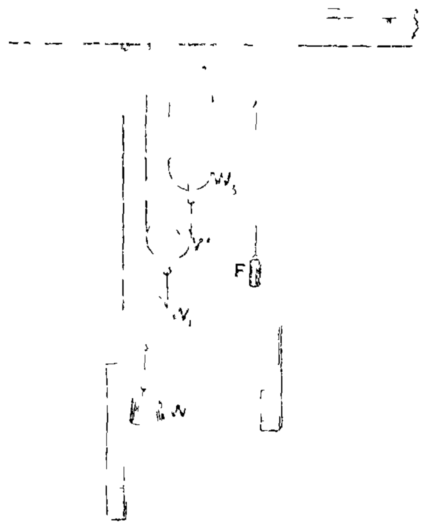

Fig 128.

W_3 are attached to a beam the string round each pulley being attached to the one above it except that round W_3 which passes over a fixed pulley A.

(a) Set up the pulleys as shown and prove by the Principle of Work that

$$\text{the Mechanical Advantage} = \frac{W}{F} = 8.$$

Lower F through 8 inches and note that W is raised 1 inch. The fixed pulley A merely deflects F.

(*b*) If w is the weight of each movable pulley, show that

$$F = \frac{W}{8} + \frac{w}{8} + \frac{w}{4} + \frac{w}{2} = \frac{W + 7w}{8}$$

$$= \frac{W + (2^n - 1)w}{2^n},$$

where n = no. of movable pulleys.

Pulley Blocks (often called *the* **2nd system** *of pulleys*)

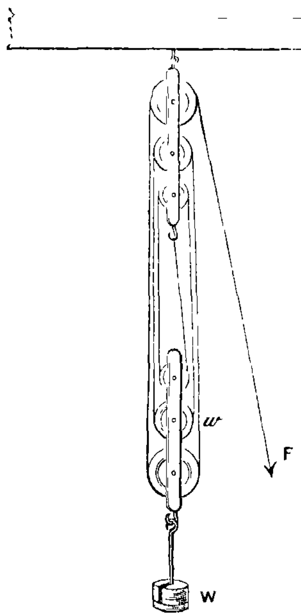

Fig. 129.

Fig 129 shows two 3-sheaved pulley blocks. A continuous string passes round the pulleys. We neglect friction and assume that the tension of the string is the same throughout the length. If an effort F supports $W + w$, it is evident that six strings all pulling with a force F support the lower block of pulleys (w) and W.

$$\therefore\ 6F = W + w,$$

$$\therefore\ \frac{W + w}{F} = 6$$

The mechanical advantage of such a system = number of strings supporting lower block.

***Exp.** Test the accuracy of this result by the Principle of Work Raise W by 1 inch and note that F, when the string is taut, has moved 6 inches.

Exercise. Make a diagram reversing the 1*st system*, *i.e.* turn Fig 128 upside down so that W_1 is attached to the ceiling and the three cords from W_1, W_2 and W_3 are supporting a beam; the pulley A is not required. **Prove** that the mechanical advantage $\frac{W}{F} = 2^n - 1$, when n = no. of pulleys (supposed weightless). This arrangement is often called "**the 3rd system of pulleys**"; it is seldom employed.

95. The Inclined Plane.

We have doubtless noticed that in order to get heavy boxes or barrels on to a dray, the carter pushes or rolls them up a slope or inclined plane, attached to the end of the dray. Thus, by using a *small force* over a *considerable distance* up the gradual slope he can raise a heavy weight.

***Exp. i. To find the mechanical advantage of an inclined plane,** the force being exerted *parallel* to the plane

Set up the apparatus provided as shown in Fig. 130. *W*, a heavy metal

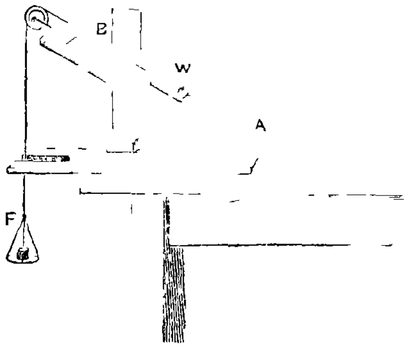

Fig 130.

roller, previously weighed, is held by a string passing over a well-oiled pulley. The plane, hinged at *A*, is gradually raised until the roller begins to move *down* the plane, and the position is noted. The plane is then gradually lowered and the position marked when the roller begins to move *up* the plane. The slope of the plane is then adjusted to the mean of these two positions and the *height* and *length* of the plane are measured.

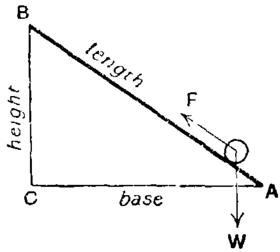

Fig. 131

Then by the Principle of Work (Fig. 131), *F* acting through a distance *AB* raises *W* a height *CB*,

i.e. work done *by F* = work done *on W*.

$F \times$ length of plane $= W \times$ height of plane,

Mechanical Advantage

$$= \frac{W}{F} = \frac{\text{length of plane}}{\text{height of plane}}.$$

***Exp. ii.** Repeat the experiment, but arrange that the **force is exerted parallel to the base** of the plane (Fig. 132). Here in raising the roller

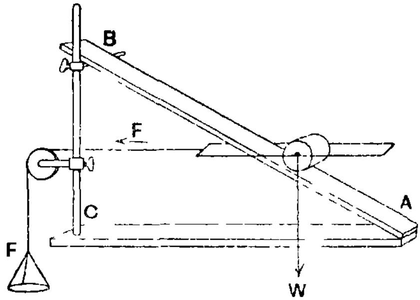

Fig. 132.

from C to B, F, the effort, being parallel to the base, would act through a distance AC, and W would be raised vertically through a height CB.

$$\therefore F \times AC = W \times CB \text{ (Principle of Work),}$$

$$\therefore \frac{W}{F} = \frac{AC}{CB},$$

$$\therefore \text{Mechanical Advantage} = \frac{\text{Load raised}}{\text{Force exerted}} = \frac{\text{base of plane}}{\text{height of plane}}.$$

96. The Screw.

Exp. Refer to § 5, Fig. 7. From a piece of paper cut out a right-angled triangle and wrap it round a pencil. As described in § 5, it is evident that, if we follow the hypotenuse of the triangle as it winds round the pencil, we are dealing with a spiral inclined plane, the *height* of which, for one turn of the screw, is the *pitch* (AK, Fig. 133), and its *base* is equal to the *circumference* ($2\pi r$).

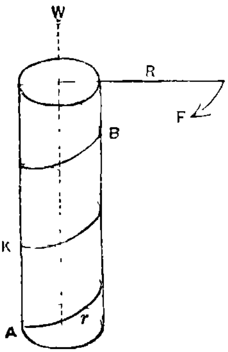

Fig. 133.

The *effort* (F) is usually applied at the end of a lever handle and acts *parallel* to the *base* of the "plane" (§ 95, Exp. ii). If R is the length of the handle, the path described by F when a complete turn is made $= 2\pi R$; at the same time the *load* (W) has been moved through AK, the *pitch* of the screw. Therefore, *theoretically*, the *mechanical advantage*

$$= \frac{2\pi R}{AK} = \frac{\text{path described by the effort}}{\text{distance moved by the end of screw}}.$$

Practically, however, friction increases as the resistance to be overcome increases and to such an extent that it is useless to perform any experiment to show the theoretical ratio between resistance and effort.

Examples XV a (Levers and Wheel and Axle).

1. A uniform lever weighing 10 lb. balances about a point 2 in. from its centre when weights 7 lb. and 10 lb. are hung from its ends. Calculate the length of the lever.

2. The diameter of the safety valve of a steam boiler is 2 in. and the distance from the centre of the valve to the hinge is 3 in. What weight must be suspended from a point on the bar 12 in. from the hinge so that the valve will just open when the pressure in the boiler is 70 lb. per sq. in.?

3. A man while sculling exerts a force of 20 lb. wt. on each scull. What is the total force on the boat if the distance from hand to rowlock is 1 ft. and from rowlock to centre of blade 4 ft.?

4. A metal cutter, AB, 2 ft. long, hinged at A, just cuts an iron bar placed 4 in. from A when a boy weighing 6 stone hangs at the end B. What pressure is exerted on the iron bar?

5. A nut which could be cracked by a weight of 15 lb. is placed 1 in. from the hinge of a pair of nutcrackers 6 in. long. What force would be needed at the ends of the handles to just crack the nut?

6. Find the effort required to raise a load of 1 cwt. if the radii of wheel and axle be 2 ft. 4 in. and 4 in. respectively.

7. What load can be lifted by a force of 20 lb. wt. if the diameters of wheel and axle be 2 ft. and 4 in. respectively?

8. It is required to lift a load of 100 lb. by means of a force of 15 lb. wt. What must be the radius of the wheel if the diameter of the axle is 6 in.?

9. Four men are raising an anchor by means of a capstan, radius of drum 8 in. and length of spoke from axis 5 ft. If each man pushes with a force of 100 lb. wt., how heavy an anchor can they raise?

10. An ordinary winch is used to raise water from a well. The bucket, weighing 25 lb., rises 11 ft., while the handle, 21 in. long, makes five revolutions. Find (*a*) the force required to turn handle, (*b*) the radius of the barrel, (*c*) the mechanical advantage of the winch.

EXAMPLES XV B (PULLEYS).

1ST SYSTEM. (*Several strings attached to beam.*)

1. With 3 movable pulleys what effort will be needed to support a load of 1 cwt.? What is the mechanical advantage of the system?

2. How many movable pulleys would be needed to raise half a ton by means of a force of 70 lb. wt.? Give the diagram, marking the tensions in the strings.

3. What load could be supported by a pull of 100 lb. wt. with five movable pulleys?

4. Find the mechanical advantage of three movable pulleys by the Principle of Work (neglecting weight of pulleys). If the power descends 8 ft. while the load rises 1 ft. what power will be required to raise 1 cwt.?

2ND SYSTEM. (*Sheaved Pulley Blocks.*)

5. What effort will support a load of 50 lb. if there are (*a*) 2 pulleys in each block, (*b*) 2 pulleys in lower and 3 in upper block?

6. What load can be raised by a force of 20 lb. wt. using two blocks each containing four pulleys, (*a*) if pulleys are weightless, (*b*) if lower block weighs 4 lb.?

7. Calculate the number of strings and weight of lower block in a system by which weights of 10 lb. and 12 lb. support loads of 38 lb. and 46 lb. respectively.

8. A man weighing 10 stone raises a load of 2 cwt. by a system of pulleys, three in each block. What will be his pressure on the ground if weight of pulleys be neglected?

9. Find by the Principle of Work the mechanical advantage of an arrangement of pulleys consisting of two blocks each containing three pulleys (supposed weightless). If the power descends 1 ft. while the load rises 2 inches what load can be raised by a power of 20 lb. wt.?

3RD SYSTEM. (*Several strings attached to weight.*)

10. What effort will be required to support 1 cwt. if three pulleys are employed supposed weightless?

11. What load can be raised by a weight of 100 lb., making use of four pulleys, supposed weightless?

12. If the load rises 1 inch, how far will the effort act vertically downwards when (*a*) two pulleys, (*b*) three pulleys, are employed?

Examples XV c (The Inclined Plane and Screw).

1. Find the force, acting parallel to the plane, required to support a load of 80 lb. on a smooth slope rising 3 in 40.

2. What force acting parallel to the plane is required to support a load of 400 gm. on an inclined plane of (*a*) 30°, (*b*) 45°, (*c*) 60°?

3. Find the horizontal force required to support a load of 100 lb. on a smooth inclined plane which has a gradient of 5 in 13.

4. A trolley mounted on frictionless wheels rests on a plane inclined at 30° to the horizontal. If the trolley weighs 20 lb., find by construction the force required to keep it in equilibrium if the cord is (*a*) parallel to the plane, (*b*) horizontal.

5. What force acting horizontally is required to support a load of 60 lb. on an inclined plane of (*a*) 30°, (*b*) 45°, (*c*) 60°?

6. Two planes 12 inches high are placed back to back. A weight of 10 lb. rests on one plane whose base is 9 inches and a string passing over a pulley at the top attaches it to a weight *W* resting on the other plane whose base is 5 inches. If there is equilibrium find the weight *W*.

7. Find the Mechanical Advantage of a screw whose diameter is 7 in. and pitch ½ in.

Examples XV d (Friction).

1. If a body weighing 40 lb. rests on a rough horizontal plane and is acted upon by a force of 8 lb. wt. acting at 60° to the plane, what is the force of friction?

2. A body resting on a rough horizontal plane is acted upon by two horizontal forces of 5 and 12 lb. wt. acting at an angle of 90°. What is the force of friction?

3. If a block of wood weighing 500 gm. lies on a rough horizontal plane, coefficient of friction 0·3, what is the least horizontal force which would move it?

4. If a force of 5 lb. wt. will just move a mass of 15 lb. on a rough horizontal plane, what is the coefficient of friction?

CHAPTER XVI.

NEWTON'S LAWS OF MOTION[1]: UNITS OF FORCE, WORK AND ENERGY.

97. First Law of Motion.

Passing reference has already been made to this law, although it was not actually named, in §§ 24, 35 and 75 (1), when the subjects of *force* and *inertia* were dealt with.

A body tends to remain in its state of rest or of uniform motion in a straight line except in so far as it is compelled by forces to change that state.

98. Second Law of Motion.

Momentum. Imagine that two boys A and B are skating on smooth ice. Let the mass of each boy be 7 stone. *Firstly*, imagine that A is standing still and that B, coming with a velocity of 8 ft. per sec., bumps into him from behind, catches hold of him and the two travel along together. We know by experience that the velocity of the two moving on together is less than B's velocity; in reality, their velocity after impact is 4 ft. per sec. Experience also tells us that if B bumped in with a velocity of 16 ft. per sec the velocity of both moving together would be greater than in the first case and that they would travel on together about twice as far before friction brought them to rest. *Secondly*, imagine that a strong man (C), whose mass is 14 stone, moving with a velocity of 8 ft. per sec., bumps into and picks up A who is standing

[1] Propounded by Sir Isaac Newton of Cambridge University in the reign of Charles II.

still, and that C carries A along with him. Again we know that the two would travel on at a greater speed and for a greater distance than when B, who weighed only 7 stone, bumped in with a velocity of 8 ft. per sec., but that C's velocity would be reduced after he had picked up A. We see therefore that the results after collision depend on the *velocity* and the *mass* of the colliding bodies.

When a hammer drives a nail into a board, the result of a well-directed blow depends (1) on the *velocity* of the hammer and (2) on its *mass*.

Again, the force required to stop a moving ship depends on the *speed* of the ship and its *tonnage*. We say colloquially that it is harder to stop an ocean liner than a sculling boat, although they may both be travelling at the same speed, because the liner "has more *way* on it." Speaking scientifically we should say that the liner's *momentum* is greater than that of the boat. **The momentum** (M) **of a moving body** depends partly on its *mass* (m) and partly on its *velocity* (v) and **is measured by the product of mass and velocity.**

$$M = mv$$

Exp. [illegible]. To explain the meaning of momentum.

Fig. 134 shows a *Ballistic Balance*. A block A and a metal ball B are hung just touching each other by vertical strings from the same beam, and their centres of gravity are at the same level: therefore as pendulums of the same length they *swing in equal periods*, and therefore the distances of their *displacements* from the vertical (amplitude, § 12) measure their *velocities* at the bottom of their swing.

Two pointers from A and B show the displacements along a scale

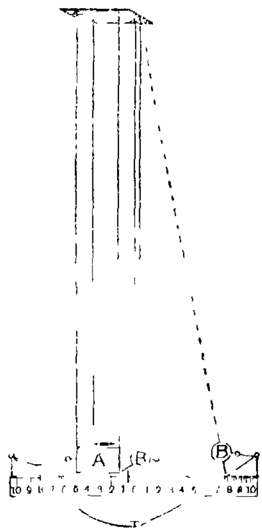

Fig. 134

Use the instrument to balance momenta.

Attach some soft wax or plasticine to the shaded surface of A so that when A and B strike each other they will stick together. Draw A and B aside by the silk threads and release them at the same moment. By careful trial adjust the displacements so that after the bodies impinge they are *brought to rest*. Obtain the *masses* of A and B $[m_A, m_B]$ by *weighing* and their *velocities* $[v_A, v_B]$ when they strike each other by measuring the *displacements* $[d_A, d_B]$

Observations show that $$\frac{m_A}{m_B}=\frac{d_B}{d_A}=\frac{v_B}{v_A},$$
$$\therefore\ m_A v_A = m_B v_B.$$

The blows must have been equal for the bodies *came to rest* after impact We therefore conclude that the measure of a blow delivered by a mass m moving with velocity v is the product mv.

mv is called the *Momentum* (M) of the moving body. It is also called the *Impulse* (I) of the blow delivered by the body when suddenly brought to rest.

Impulse. The word *impulse* is used of a force applied or communicated *suddenly*. But however sudden the application of the force, the delivery of the blow must occupy *time* Refer to § 75 (3). We learnt that the longer a force acts on a body the greater the body's velocity becomes. The trolley (§ 75) starting from rest and pulled by a constant force moved with a constant acceleration. Its *momentum* (mv) increased as the *time* during which the *force* was acting increased. Hence the product of *force* and *time* is a measure of the *momentum* generated in a body[1].

$$\text{Momentum} = \text{force} \times \text{time}.$$

The product of *force* and *time* during which the force acts is called the **Impulse** (I) of the force. Hence

$$M = mv = Ft = I.$$

[1] Assuming Newton's Second Law and the relation $F=mf$ [see *infra*],

since $\qquad \text{Momentum} = mv$;

and $\qquad v = ft$ (if the body starts from rest),

$\therefore$ Momentum $= mft$; but $F=mf$,

$\therefore$ **Momentum** $= Ft$

Exp. ii. Let us use the Ballistic Balance to show that the momentum generated is proportional to the impulse causing motion.

N.B. The sticky substance must be completely removed from both A and B.

(1) A is stationary and B is drawn aside 4 divisions by a fine silk thread and released by burning the thread. B strikes A and the *impulse* of the blow imparts *momentum* to A. A's *velocity* at the moment of *impact* (*i.e.* when the blow was struck) is measured by the distance (d) that A's pointer moves along the scale. The mean of several observations is taken.

(2) The experiment is repeated but B is drawn aside 8 divisions; it therefore strikes A with *doubled* velocity. The *impulse* is therefore doubled and A moves along the scale a distance $2d$.

Vary the experiment (a) by replacing B with another ball of twice the mass, (b) by placing a tight fitting metal cylinder in the hole in A by which the mass of A may be increased (say doubled).

The results of the Trolley experiments (§ 75) combined with our experiments with the Ballistic Balance help us to understand the relation between impressed *force* and *momentum*.

From observations in the Trolley experiments, using *any* units we chose, we concluded that

$$\text{Force} \propto \text{mass} \times \text{acceleration},$$

i.e. $$\text{,,} \propto m \times \text{rate of change of } v,$$

and, since m is constant,

$$\text{Force} \propto \text{rate of change of } mv,$$

∴ *impressed force is proportional to rate of change of momentum.*

This is Newton's

Second Law of Motion. Rate of change of momentum is proportional to the force causing motion and takes place in the direction of this force.

We have endeavoured to arrive at Newton's Second Law by experiment; if however we start with the assumption that this Law is true, we may write immediately

Force $\propto$ rate of change of momentum,

$\propto$ rate of change of mv,

$\propto m \times$ rate of change of v [m being constant],

$\therefore \; F \propto mf.$

If we choose our **unit of force** to be such that acting on *unit mass* it produces *unit of acceleration*, we can write

Force = mass × acceleration,

$$F = mf.$$

Absolute Unit of Force. Under the British system, the **absolute unit of force** is called a **poundal** and is such that acting on a mass of one pound for one sec. it increases the velocity by 1 ft. per sec., *i.e.* it generates in a mass of one pound an acceleration of 1 ft. per sec. per sec.

But the **weight of a pound** acting on a *pound mass* generates an acceleration of g ft. per sec. per sec. (see § 73).

$\therefore$ **1 pound weight** = g **poundals.** [$g = 32$ approx.]

Under the Metric System, the absolute unit of force is called a **dyne,** which generates in a mass of 1 gram an acceleration of 1 cm. per sec. per sec.

But the **weight of a gram** acting on a *gram mass* generates an acceleration of g cm. per sec. per sec.

$\therefore$ **1 gram weight** = g **dynes.** [$g = 981$ approx.]

N.B. 1. Since the value of g varies for different places on the earth, the *pound* and the *gram* are *not absolute* units of force.

2. The Formula, $F = mf$, is only true where F is measured in absolute units, hence, in using this formula, pounds weight and grams weight must be converted to poundals and dynes respectively by multiplying by g or g.

99. Third Law of Motion.

To every action there is an equal and contrary reaction.

We see a good example of this law when two skaters on hard smooth ice try to push each other in opposite directions: each moves away from a fixed point on the ice between them.

Again, the same result is shown when a man in a boat pushes away from another boat—both boats move in opposite directions. The case is not so evident if he pushed away from a large ship, nevertheless he would move the ship slightly away from his boat. If the man jumps from his boat into the water, he pushes the boat away from him in the opposite direction to that in which he is jumping.

The reverse of these movements is seen if the man in the boat hauls a rope attached to another boat; as he shortens the line both boats approach each other. Again, if a piece of soft iron and a magnet each on a separate cork are floated in a bowl of water, they are mutually attracted to each other and both move through the water until they touch.

When a gun is discharged the gun recoils with the same *momentum* as the shot but in the opposite direction.

See also § 51 and recall many other instances of similar mutual reactions.

100. Practical Units of Work.

In § 90 we learnt that (1) a force does work when it moves its point of application and (2) work done = force × distance through which the point of application is moved in the direction in which the force acts.

Engineers use the *Gravitation Unit of Work* which is called a **foot-pound.** This measures the *work done* by a force equal to the weight of a *pound* when it moves its point of application through *one foot* measured in the direction of the force, *or*, the work done in lifting one pound mass vertically through one foot.

Ex. What work is done by a man rolling a barrel weighing 150 pounds up a slope 60 feet long rising 1 in 10?

The work is done against gravity which acts vertically. Since the slope

rises 1 ft. in 10 ft., in rolling the barrel 60 ft. up the slope the barrel will be raised 6 ft. against gravity.

$\therefore$ **Work done** $=150\times 6=$ **900 foot-pounds.**

The *kilogram-metre* unit of work is done by a force equal to the weight of one kilogram acting through one metre.

Absolute Units of Work.

Foot-poundal

= work done by a force of one poundal acting through 1 ft.

Dyne-centimetre

= work done by a force of one dyne acting through 1 cm.

= one **erg.**

101. Power is rate of doing work. The unit of *time* is introduced. In practical units, an engine working at **one horse-power** (H. P.) does 33,000 foot-pounds in one minute, *or*, it would raise a mass of 550 pounds 1 ft. vertically per second The **absolute unit of power** is an **erg** per **second.**

Energy is **capacity to do work** and is therefore measured in the same units as work—viz. in foot-pounds, foot-poundals and dyne-centimetres (ergs).

Potential Energy or energy of position. Water stored in a reservoir on a mountain can do work in virtue of its *position*: the water in pipes leading from the reservoir is at pressures depending upon the vertical distance below the surface of the water in the reservoir; this pressure applied to a turbine or water wheel moves its point of application and work is done.

Kinetic Energy or energy of motion. A moving body can do work in virtue of its *motion*: its energy is measured by the work it can do before being brought to rest, which equals the work done *on* the body in bringing it to its condition of motion.

To find the Kinetic Energy of a body of mass m moving with a velocity v.

Let the body start from rest ($u=0$) and acquire a velocity v in a distance s. Let it be acted on by a constant force of F poundals (or dynes) producing an acceleration f. Then

$$Fs = mf \times s,$$

but $$v^2 = u^2 + 2fs; \quad \therefore fs = \tfrac{1}{2}v^2,$$

$$\therefore Fs = \tfrac{1}{2}mv^2,$$

$\therefore$ the Kinetic Energy[1] of a moving body

= **half the product of its mass and the square of its velocity.**

102. Newton's Law of Gravitation[2]. Every particle of matter in the universe attracts every other particle with a force which is proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$G \propto \frac{m_1 m_2}{d^2},$$

where G is the force of attraction between two masses m_1 and m_2 at a distance of d from each other.

EXAMPLES XVI A (LAWS OF MOTION).

($g = 32$ ft./sec.^{2} or 981 cm./sec.^{2})

1. What acceleration is produced by a force of (*a*) 20 pdls., (*b*) 20 lb. wt., acting on a mass of 10 lb.?

2. What force in lb. wt. will produce an acceleration 20 ft. sec. units in a mass of 40 lb.?

3. A force of 10 lb. wt. produces an acceleration of 5 ft. sec. units in a certain body. Calculate the mass of the body.

4. What force acting upon a mass of 10 lb. for 5 seconds will produce a velocity of 40 ft. per sec.?

5. A mass of 40 lb. is acted on by a force of 5 lb. wt. What distance will it traverse in 6 sec.?

6. Calculate the number of dynes in one poundal. (1 lb. = 453·6 gm.)

7. What acceleration is produced by a force of 40 dynes acting on a mass of 1 decagram?

8. A force of 100 dynes acts upon a mass of 50 gm. for 10 seconds. What velocity does it produce?

[1] In *absolute* units, *i.e.* in foot-poundals or in ergs.

[2] First formulated about the year 1684.

9. A force of 10 gm. wt. acts upon a mass of 100 gm. What acceleration is produced?

10. If a force of 5 gm. wt. acts on a mass of 10 gm. for 4 seconds, how far does the mass move?

11. A mass of 10 lb. moving with a velocity of 20 ft. per sec. is brought to rest by a constant resistance after traversing 100 ft. Calculate the resistance in (*a*) poundals and (*b*) pounds weight.

12. What force in lb. wt. is needed to stop a train of 100 tons mass travelling at 60 miles per hour (*a*) in 352 yd., (*b*) in 44 sec.?

EXAMPLES XVI B (MOMENTUM AND IMPULSE).

1. Calculate the momentum of

   (*a*) a mass of 10 lb. moving with a velocity of 5 yd. per sec.,

   (*b*) a cricket ball ($5\frac{1}{2}$ oz.) moving with a velocity of 32 ft. per sec.,

   (*c*) a train of 100 tons travelling at 60 miles per hour.

2. A $\frac{1}{4}$ ton shot is discharged from an 80 ton gun with a velocity of 1600 ft. per sec. What is the gun's velocity of recoil?

3. A body of mass 3 lb. is acted on by a force which changes its velocity from 10 miles to 20 miles per hour. What is the impulse of the force?

4. A cricket ball ($5\frac{1}{2}$ oz.) lying on the ground is struck by a bat and starts off with a velocity of 16 ft. per sec.

   If the ball remained in contact with the bat $\frac{1}{100}$ sec., find

   (*a*) the average force acting on the ball,

   (*b*) the impulse of this force.

5. A batsman blocks a ball ($5\frac{1}{2}$ oz.) which is shooting straight towards his wicket with a velocity of 20 ft. per sec. If the ball starts back to the bowler with a velocity of 16 ft. per sec. what is (*a*) the change of momentum of the ball, (*b*) the average force exerted on the ball, assuming that the bat and ball remain in contact $\frac{1}{10}$ sec.?

6. Two boys, *A* 8 stone, *B* 9 stone, while playing football charge one another and are brought to rest. If *A* was moving at 7 miles per hour what was *B*'s velocity?

7. A body of mass 5 lb. moving with a velocity of 20 ft. per sec. overtakes a body of mass 4 lb. moving with a velocity of 5 ft. per sec. in the same straight line. If they move on together find the new velocity. What would the new velocity be if the bodies had been moving in opposite directions at the time of impact?

Examples XVI c (Energy).

1. A particle of mass 10 lb. is moving with a velocity of 20 ft. per sec. Calculate its Kinetic Energy, stating your units.

2. Find the Kinetic Energy of a ball weighing 1 Dgm. and moving with a velocity of 10 cm. per sec.
In what units is your answer?

3. A mass of 5 lb. falls through a vertical distance of 100 ft. What is its Kinetic Energy (*a*) in ft. poundals, (*b*) in ft. pounds?

4. A train of mass 100 tons is moving at 60 miles per hour. If the brakes stop it, when steam is cut off, in $\frac{1}{2}$ mile, what is the resistance in pounds per ton?

5. A cricket ball ($5\frac{1}{2}$ oz.) is moving at the rate of 10 ft. per sec. What force will be required to stop the ball if the fielder moves his hand back (*a*) 1 ft., (*b*) 1 in.?

REVISION QUESTIONS FROM OXFORD AND CAMBRIDGE LOCAL EXAMINATION PAPERS.

PAPER A.

1. A body situated at A has imparted to it simultaneously velocities of 9 and 10 ft. per sec. respectively along straight lines AB and AC which are inclined to one another at an angle of 37°. Find by means of an accurate diagram the magnitude of the velocity with which the body begins to move, and the angle between this direction and AB.

2. A particle moving in a right line with uniform acceleration has at a given instant a speed of 5 ft. per sec., and 10 sec. afterwards it has a speed of 35 ft. per sec. What is the magnitude of the acceleration? Over what distance does the particle move in the interval?

3. State Newton's Three Laws of Motion and illustrate the first and third by any two examples.

What acceleration is produced in a mass of 2 lb. by a force equal to the weight of 6 oz.?

4. A particle projected vertically upwards from a point A with a velocity of 60 ft. per sec. passes through a point B which is 36 ft. above A. What interval of time will elapse before the particle passes again through B?

5. Explain the meaning of the terms *acceleration*, *dyne*, *erg*.

A particle moves in a straight line for 4 sec. with a constant speed of 8″ per sec. and then the motion is uniformly accelerated at the rate of 2″ per sec. per sec. Find the whole distance travelled at the end of 10 sec.

6. Distinguish clearly between the *mass* of a body and its *weight*.

7. Give a definition of each of the terms *force*, *momentum*, *work*, *energy*. If a mass of 5 lb. has a velocity of 16 ft. per sec. how much work should be done on it to stop it?

8. A ball is dropped from a stationary balloon at a height of 1600 ft.: 5 sec. later a shot is fired vertically at it from below with a velocity of 1040 ft. per sec. Find when and at what height the shot hits the ball.

9. A mass of 3 gm. has a velocity of 60 ft. per sec., and a mass of 10 gm. has a velocity of 20 ft. per sec.

What is the ratio (*a*) of their momenta, (*b*) of their Kinetic Energy?

10. Assuming that g is 981 cm. per sec. per sec., prove that a dyne is roughly equal to the weight of a milligram.

Paper B.

1. Define the moment of a force about a given point and state how you would distinguish between positive and negative moments.

AB is a horizontal lever movable about B and 4 ft. long. What vertical resistance at C (18″ from B) will be overcome by an upward force of 30 lb. wt. applied at A?

2. Draw a sketch of a wheel and axle. If the radius of the wheel is 2 ft. and that of the axle 9 in., what force will be required to lift a load of 240 lb.?

3. Two unlike parallel forces have magnitudes 26 and 12 gm. wt. and the perpendicular distance between their lines of action is 7 cm. Draw the figure to scale and find the magnitude and line of action of the resultant.

4. There is a circular piece of cardboard of radius 8 cm. A circular disc of radius 6 cm. is cut out, the two circles touching one another; find the distance of the centre of gravity of the remainder from the centre of gravity of the original disc.

5. State what is meant by *components* of a force.

There is a force of 13 lb. wt. and it is to be replaced by two whose magnitudes are 12 and 5. Taking any convenient unit to represent a lb. wt. make the resolution by means of your instruments. Can the force 13 be replaced by forces of 8 and 4? Give a reason for your answer.

6. A uniform bar 4 ft. long and weighing 12 lb. is placed over a peg D and rests in a horizontal position with one end under another peg C. If the distance from C to D is 18″ draw a diagram showing the action of the forces on the bar and determine the pressure on the pegs.

7. State the Principle of Work as applied to a machine.

Show how it gives the relation between the forces which will produce equilibrium when applied to the ends of the arms of a lever.

8. Find the horizontal force required to support a weight of 10 lb. on a smooth inclined plane whose inclination to the horizon is 30°.

9. A particle A of mass 2 oz. is suspended from two points B and C in the same horizontal line by two strings AB and AC; the angle BAC is a right angle, $AB=3$ in. and $AC=4$ in. Draw an accurate figure and produce CA to meet the vertical through B in D. Show that DAB is a Triangle of Forces for the weight and the tensions in the strings and hence by measurement find the tensions.

10. Draw an equilateral triangle ABC with a side of 4 cm. and let the following forces act along its sides: 10 from A to B, 15 from B to C and 5 from C to A. Find the magnitude and line of action of the resultant.

SECTION IV.

HEAT.

CHAPTER XVII.

HEAT, TEMPERATURE, THERMOMETRY.

103. What is Heat?

If we rub a brass button rapidly on a piece of wood, the button becomes so hot that we cannot hold it. When we pump up a bicycle tyre, we are doing *work* in compressing the air into the far end of the pump and we notice that *heat* is developed. Primitive races are said to obtain fire by rapidly rotating a wooden stick in a hole in another piece of wood. The energy of a railway train in being brought to rest by the action of the brakes is largely dissipated in the form of heat; the brakes and rails become hot and sparks fly from the wheels when pressure and friction are applied. It is evident then that (1) heat may be developed by work, and (2) the energy of a moving body may be converted into heat.

On the other hand heat may be converted to *potential energy*, as for instance when the furnace converts water to steam and the latter is stored at high pressure in a boiler. The *potential energy* of the compressed steam is changed into *kinetic energy* when this steam is released to force the piston backwards and forwards in the cylinder of an engine.

Heat then is simply a *form of energy*, a "mode of motion" as it has been termed.

Temperature. Heat can be transferred from one body to

another. The body which *receives* heat from another body is said to be at a *lower temperature*; the body which *gives* heat is said to be at the *higher temperature*. *Temperature* therefore is a *condition* which determines whether the body shall *receive* or *give* heat to another body.

*104. Exp. To show the difference between heat and temperature.

A piece of iron (say a *seven-pound weight*) is heated until it is too hot to touch more than momentarily but by no means red hot. A tin-tack is heated to *redness* in the Bunsen flame. The tin-tack on being placed on the iron weight *gives* up some of its heat to the iron and soon may be touched momentarily in contact with the iron without burning the hand. The tin-tack when red hot was at a *higher temperature*, but did it contain more *heat*?

Fill a pint can half full of cold water. Heat the tin-tack red hot and drop it into the water. The water still remains cold. Next place the hot iron weight in the water. In a few seconds the water is appreciably warmer. It is evident that although the weight was at a *lower temperature* than the tin-tack before it was placed in the water, yet the weight contained a greater quantity of heat.

We see then that the quantity of heat which a body contains depends partly on its *temperature* and partly on its *mass*. If the tin-tack were of iron and had weighed seven pounds, it would, when red hot, have not only been at a higher temperature than the weight but it would also have contained more heat, as would have been shown by its power of raising the half-pint of water to a higher temperature than the weight did.

Later we shall find by experiment that equal weights of *different* substances at the same temperature hold different quantities of heat. For instance, 7 lb. of aluminium would have heated the half-pint of water to a much greater extent than the 7 lb. of iron did, although before dropping the metals in the water they were both at the same temperature.

105. Thermometers.

Thermometry is the division of the subject which treats of the accurate measuring and recording of temperature. The instrument used is called a **Thermometer.**

***Exp. i. To make a model of a liquid thermometer.**

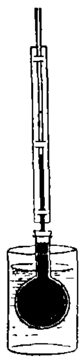

Fig. 135.

Fit a rubber cork, through the hole of which a long glass tube has been pushed flush with its end, into a 250 c.c. flask filled with cold water and coloured with indigo (Fig. 135). Place a cardboard scale behind the tube and mark the top of the column of water. This piece of apparatus constitutes a rough model of a thermometer. Place the flask in hot water and notice that the liquid immediately descends. The reason for this is that the glass expands with the heat and therefore the flask becomes larger and more liquid is required to fill it. But soon the heat passes through the glass into the water and the column rises steadily. Evidently the water expands more than the glass. Take the flask out of the hot liquid and put it under a stream of cold water. Note that the column at first rises and then falls again. (1) Replace the tube by a *finer* tube and note that the instrument is much more sensitive. Explain the reason for this increased sensitiveness. (2) Replace the flask by a smaller one and note the decreased sensitiveness.

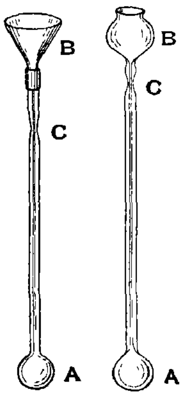

Fig. 136.

***Exp. ii. To make an alcohol thermometer.**

You are provided with a piece of capillary tubing at one end of which *A* is a bulb (Fig. 136) and at the other *B* a small funnel. The tube is constricted at *C*. Pour alcohol (coloured pink with magenta) into *B*. Have ready a beaker of boiling water. Holding the tube in a vertical position, bring it over the boiling water. Bubbles of air pass up from the bulb through the alcohol, thus showing that air expands on being heated. Remove the tube from the source of heat so that the air inside may cool. Contraction of the air takes place and the alcohol runs down into the bulb. Boil the alcohol in *A* by placing *A* in the hot water. The air is entirely displaced. Repeat the cooling and heating until finally the bulb and tube are completely filled with alcohol. While the alcohol is still hot the

tube may be sealed[1] off at *C* by softening the glass in the blowpipe flame and then drawing it out while still in the flame (see *Exp. Sci., Chem.* § 3).

Mercury is generally used in thermometers for the following reasons:

(1) It can be used through a large range at ordinary temperatures (b.p. 357° C., f.p. – 39° C.).

(2) It increases and decreases in volume uniformly for equal rises and falls of temperature.

(3) It conducts heat well and soon reaches the same temperature throughout.

(4) It does not require much heat to raise its temperature (its capacity for heat is low).

(5) It is opaque and does not wet glass.

(6) It does not readily evaporate.

Alcohol is useful for measuring *low temperatures* (f.p. – 130° C., b.p. 78° C.).

106. The fixed points. The *freezing point* and the *boiling point* of water are the two fixed points from which a scale of temperatures is arranged. The freezing point of water does not vary under ordinary conditions but the boiling point changes with the *height* of the *barometer*. The temperature of the steam from pure water boiling when the barometer stands at 760 mm. is taken as the higher of the two fixed points.

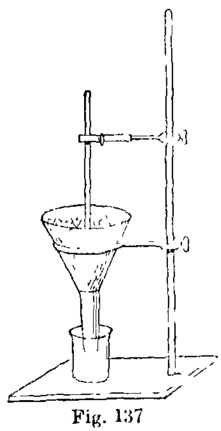
Fig. 137

The **freezing point** is determined by placing the thermometer in snow or powdered ice which is allowed to melt slowly, but the water formed is drained away (Fig. 137). A mark is made on the stem to show the top of the column of liquid when the bulb is in melting ice.

[1] In sealing off the tube, the temperature of the bath should be somewhat above the highest temperature which the thermometer is required to register.

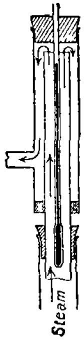

Fig. 138.

The **boiling point** is fixed by placing the mercury thermometer in a current of steam in a tube surrounded by a steam jacket (Fig. 138). The instrument is called a *hypsometer*.

The barometer should register 760 mm. pressure. The thermometer tube is drawn out through the cork until the top of the column is above the cork; a mark is then made on the stem to show the boiling point.

Graduation. The distance between F.P. and B.P. on the tube is divided in three different ways, according to the three scales shown in Fig. 139 and in the following table:

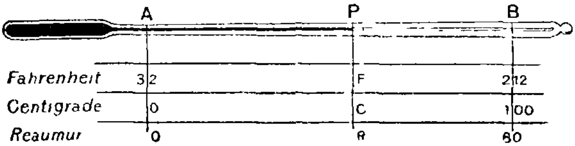

Fig. 139.

Scale	Symbol	Freezing Point (F.P.)	Boiling Point (B.P.)	No. of Divisions between F.P. & B.P.	Proportional (in last col.)
Fahrenheit	*F*	32°	212°	180	9
Centigrade	*C*	0°	100°	100	5
Réaumur	*R*	0°	80°	80	4

In order that the reading on *all* the scales may be reckoned from a freezing point at 0°, we must subtract 32 from the Fahrenheit readings, then, if *F*, *C* and *R* stand for the corresponding readings on the three scales, we have the equation—

$$F - 32 : C : R = 180 : 100 : 80 = 9 : 5 : 4,$$

e g.

$$\frac{F-32}{C} = \frac{180}{100} = \frac{9}{5}.$$

Ex. i. What is the temp on the C. scale corresponding to 59° F ?

$$\frac{59-32}{C} = \frac{9}{5}, \quad \therefore C = \frac{27 \times 5}{9} = 15° \text{ C.}$$

Ex. ii. What is the temp. on the F scale corresponding to 80 C.?

$$\frac{F-32}{80} = \frac{9}{5}, \qquad F = 32 + \frac{80 \times 9}{5} = 176° \text{ F.}$$

Ex. iii. Knowing that 0° C corresponds to 32° F.,
and that 15° C. „ 59° F.,
construct a graph to show the corresponding readings in the two scales (see Fig. 140)

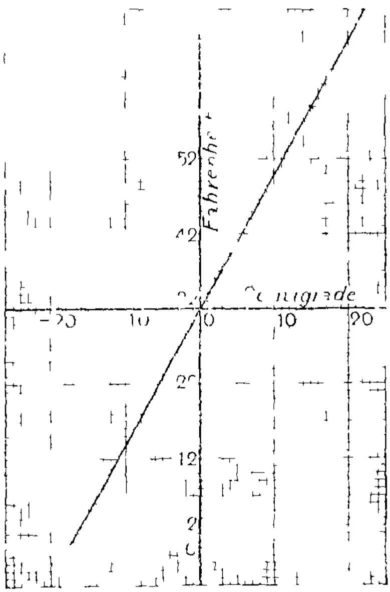

Fig 140

The *Centigrade* scale is used for scientific purposes and is largely in vogue throughout the Continent. The Fahrenheit scale is used chiefly in British territory.

***Practical Exercises.**

Test the F.P. of the various thermometers provided, using melting ice and a funnel (Fig. 137).

N.B. If the top of the column gives a reading *above* the fixed point on the scale the correction must be entered as negative, if *below* as positive. (For *Zero error*, see §§ 6 and 7.)

107. Maximum and Minimum Thermometers.

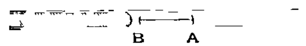

Fig. 141.

Often we require to know the highest and the lowest temperatures reached during an interval.

Maximum thermometers register the highest temperature attained. The thermometer in Fig. 141 is placed in a *horizontal* position. *Mercury* is the liquid used. A small float of glass or steel *AB* in the shape of a dumb-bell is placed inside the tube before it is sealed up. This float is pushed in front of the mercury and is left stranded when the mercury retreats. To set the instrument the thermometer is tilted until the float touches the mercury column.

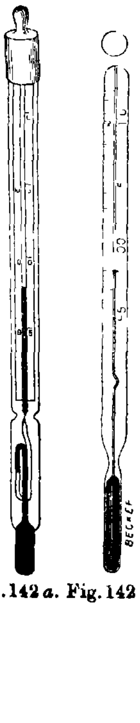
Fig. 142 *a*. Fig. 142 *b*.

The **clinical thermometer** is a maximum thermometer arranged to give readings of temperatures from about 96° to 110° F. It is used by medical men to test the temperature of the body. Man's normal temperature is 98°·4 F. A small constriction in the tube causes the column of mercury to break so that when it has reached the maximum reading a small thread is left stranded. To set the thermometer the thread must be shaken back towards the bulb (Figs. 142 *a* and *b*).

Minimum thermometers usually contain *alcohol* as the expanding and contracting liquid. A glass index AB (Fig. 143) inside the alcohol is carried back to the lowest point registered, as the surface tension of the alcohol at A is sufficient to keep the meniscus (A) intact: on the temperature rising the alcohol flows past the stranded float. To set the thermometer the instrument is tilted until the float touches the meniscus at A.

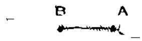

Fig. 143.

In **Six's Thermometer** we have a maximum and minimum instrument combined (Fig. 144). The bulb A and the tube AC contain alcohol. From C to D the tube contains mercury and from D to B alcohol, the bulb B contains alcohol and alcohol vapour and air under pressure, which prevents the thread of liquid from breaking. Small steel floats are pushed by the mercury as it advances in the tube DB (maximum) when the temperature rises, or as it retreats in the tube AC (minimum) when the temperature falls. A fine hair spring is attached to each float and holds it in position by pressure against the inside of the tube. To set the instrument a magnet is used to bring the floats down until they touch the mercury. If the instrument is accurately graduated the ends of the mercury column should always give the same readings on the scales.

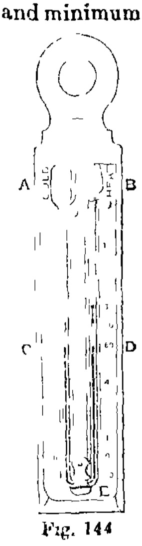

Fig. 144

EXAMPLES XVII (THERMOMETERS).

1. The position of the mercury in an ungraduated thermometer is marked when the instrument is placed in ice and steam respectively. The distance between these marks is found to be 15 cm. When the thermometer is suspended in the laboratory the mercury is found to stand 2·4 cm. above the lower mark. What is the temperature of the laboratory on (*a*) the Centigrade scale, (*b*) the Fahrenheit scale?

The *Centigrade* scale is used for scientific purposes and is largely in vogue throughout the Continent. The Fahrenheit scale is used chiefly in British territory.

***Practical Exercises.**

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N.B. If the top of the column gives a reading *above* the fixed point on the scale the correction must be entered as negative, if *below* as positive. (For *Zero error*, see §§ 6 and 7.)

107. Maximum and Minimum Thermometers.

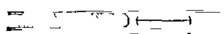

Fig. 141.

Often we require to know the highest and the lowest temperatures reached during an interval.

Maximum thermometers register the highest temperature attained. The thermometer in Fig. 141 is placed in a *horizontal* position. *Mercury* is the liquid used. A small float of glass or steel *AB* in the shape of a dumb bell is placed inside the tube before it is sealed up. This float is pushed in front of the mercury and is left stranded when the mercury retreats. To set the instrument the thermometer is tilted until the float touches the mercury column.

Fig. 142 *a*. Fig. 142 *b*.

The **clinical thermometer** is a maximum thermometer arranged to give readings of temperatures from about 96° to 110° F. It is used by medical men to test the temperature of the body. Man's normal temperature is 98°4 F. A small constriction in the tube causes the column of mercury to break so that when it has reached the maximum reading a small thread is left stranded. To set the thermometer the thread must be shaken back towards the bulb (Figs. 142 *a* and *b*).

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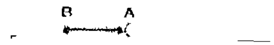


Fig 143

the surface tension of the alcohol at A is sufficient to keep the meniscus (A) intact· on the temperature rising the alcohol flows past the stranded float To set the thermometer the instrument is tilted until the float touches the meniscus at A.

In **Six's Thermometer** we have a maximum and minimum instrument combined (Fig. 144). The bulb A and the tube AC contain alcohol. From C to D the tube contains mercury and from D to B alcohol, the bulb B contains alcohol and alcohol vapour and air under pressure, which prevents the thread of liquid from breaking. Small steel floats are pushed by the mercury as it advances in the tube DB (maximum) when the temperature rises, or as it retreats in the tube AC (minimum) when the temperature falls A fine hair spring is attached to each float and holds it in position by pressure against the inside of the tube. To set the instrument a magnet is used to bring the floats down until they touch the mercury. If the instrument is accurately graduated the ends of the mercury column should always give the same readings on the scales.

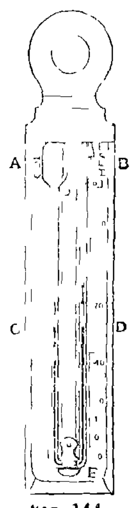


Fig 144.

EXAMPLES XVII (THERMOMETERS).

1. The position of the mercury in an ungraduated thermometer is marked when the instrument is placed in ice and steam respectively. The distance between these marks is found to be 15 cm. When the thermometer is suspended in the laboratory the mercury is found to stand 2·4 cm. above the lower mark. What is the temperature of the laboratory on (*a*) the Centigrade scale, (*b*) the Fahrenheit scale?

2. How many degrees will the mercury in a Fahrenheit thermometer rise if a Centigrade thermometer registers a rise of 15°?

3. Convert to Fahrenheit: 15° C.; – 20° C.: 40° C.: 2° C.

4. A thermometer is graduated to read 5° in ice and 75° in steam What Centigrade temperature corresponds to 33° on this thermometer?

5. At what temperature will Centigrade and Fahrenheit thermometers read the same?

6. How many degrees will the mercury in a Centigrade thermometer fall if the temperature of the room sinks 4·5° F.?

7. Convert to Centigrade: 68° F.; 23° F.: 194° F.: – 13° F.

8. A thermometer reads – 5° in ice and 85° in steam. What will it register when placed in a liquid at a temperature of 50° C.?

9. At what temperature will the Centigrade reading be half that of the Fahrenheit?

10. Mercury boils at 357° C. and solidifies at – 39° C. Find the corresponding Fahrenheit temperatures.

11. A thermometer is graduated to read – 20° in ice and 80° in steam. What temperature on this thermometer corresponds to 14° F.?

12. At what temperature will the Centigrade reading be one-fifth the Fahrenheit?

13. Alcohol solidifies at – 202° F. and boils at 172·9° F. Calculate the corresponding Centigrade temperatures.

CHAPTER XVIII.

EXPANSION OF SOLIDS.

108. In every-day life we are acquainted with the fact that solids expand and contract with the ordinary rise and fall of temperature due to the varying seasons. The girders of bridges and the metal rails on the railway lines are laid with a small space between the ends to prevent "buckling" if the temperature rises considerably. The wooden felloes of a wheel are bound tightly together by the metal tyre which is put on in a red hot condition. Water is thrown on the heated metal rim which cools, contracting with immense force.

***Exp. To show that a metal bar expands on heating and contracts on cooling.**

Fig. 145 shows a flat bar *AB* supported on two blocks and fixed at *A*. The end *B* rests on a knitting needle to which is attached a light pointer.

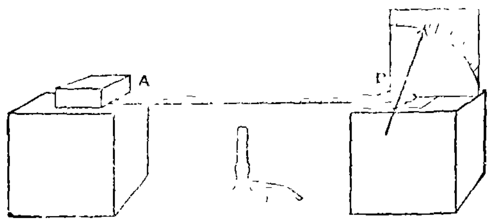


Fig. 145.

On heating the bar the end *B* moves and turns the knitting needle through an angle which is indicated by the motion of the pointer. On removing the flame the bar contracts and the needle turns back to its original position.

Expansion and contraction affect the *length*, *area* and *volume* of a *solid*. With reference to *liquids* and *gases* we can only speak of expansion and contraction as affecting their *volume*. We can therefore speak of the *linear*, *superficial* and *cubical* expansion of a *solid*, but of *liquids* and *gases* the term expansion is used only with reference to *cubical expansion*.

Coefficient of Expansion. The fraction which unit quantity (of *length*, *surface* or *volume*) of a substance expands on being heated through 1° C. is called the **coefficient of expansion** (*i.e.* of length, surface or volume as the case may be).

109. Coefficient of Linear Expansion (of Solids).

These fractions are exceedingly small, as we see from the following table:

Aluminium	·000023	Steel	·000011
Brass	·000019	Zinc	·00003
Copper	·000017	Glass	·0000086
Iron	·000012	Platinum	·0000086
Silver	·000019		

For instance, a foot of steel becomes 1·000011 ft. on being heated 1° C., *i.e.* it lengthens itself by about $\frac{1}{100000}$ of its original length.

If a = the coefficient of linear expansion of the solid,

Unit length becomes $1 + a$ when the temp. is raised 1° C.,

and becomes $1 + 2a$ „ „ „ 2° C.,

$1 + at$ „ „ „ t° C.,

and therefore

L_0 units of length become $L_0(1 + at)$ when the temp. is raised t° C., where L_0 = length of L units at 0° C.

If L_t = length when the temp has been raised t° C.,

$$L_t = L_0(1 + at).$$

Coefficient of Superficial Expansion (= 2a).

1 foot (length) becomes $(1 + a)$ ft. (length) when temp. is raised 1° C.,

$\therefore$ 1 square foot becomes $(1+a)^2$ sq ft. when temp is raised 1° C.

$$= 1 + 2a + a^2,$$

i.e. if DA expands AA' on being heated 1° C. (Fig. 146), then square $ABCD$ becomes square $ABCD + 2ABEA' + BFB'E$, but as a is a very small fraction, a^2 is negligible, hence the coefficient of superficial expansion $= 2a$.

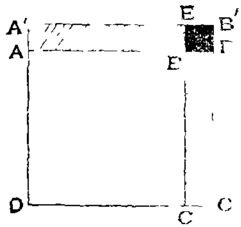


Fig. 146.

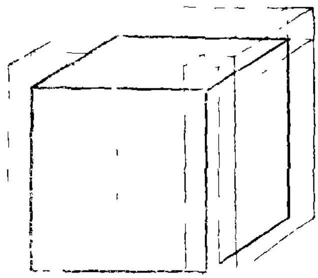


Fig. 147.

Coefficient of Cubical Expansion ($= 3a$).

Similarly

1 cu ft. becomes $(1+a)^3$ on being heated through 1° C.,

i.e.

1 cu. ft. becomes $1 + 3a + 3a^2 + a^3$ on being heated through 1° C.,

$3a^2 + a^3$ are negligible, since a is a small fraction,

$\therefore$ the coefficient of cubical expansion $= 3a$.

Ex. i. Fig. 147 is a diagram of the amount of expansion of a small cube of unit edge when heated.

Copy Fig. 147 and mark which parts correspond to 1, $3a$, $3a^2$ and a^3.

Ex. ii. Given that the coefficient of linear expansion of steel $= \cdot000011$, find the *length, area of face* and *volume* of a steel cube 7 feet long, at 15° C., when the temperature is raised to 65° C.

The rise in temp $= 65 - 15 = 50°$ C.

Each *edge* becomes $7(1+\overline{\cdot 000011\times 50})$ ft.,
,, *face* ,, $49(1+\overline{2\times\cdot 000011\times 50})$ sq. ft.,
and the *volume* ,, $343(1+\overline{3\times\cdot 000011\times 50})$ cu. ft.

110. To find the coefficient of linear expansion (*lever* method).

*Exp. i. Fig. 148 represents an inexpensive yet effective form of apparatus. A long glass tube (about 80 cm.) has a constriction at L and a small hole at M. The wire of metal (say *aluminium*) whose coefficient of expansion is required, knotted at L passes through M and is hooked through a hole B in the lever ABD. The lever[1] ABD (of brass sheet) is very accurately made, the centres of two small holes A and B being drilled *exactly* 0·5 *cm.* apart; the length of the lever from A to D is (conveniently) 20 cm. The lever is pivoted at A, a fine needle driven into a raised piece of hard wood attached to the board. A weight W keeps the wire taut. Let the temp. of the room be 15° C. Note the position of D on the scale S. Pass steam at 100° C. through the tube and when D has ceased to fall measure the *drop*,

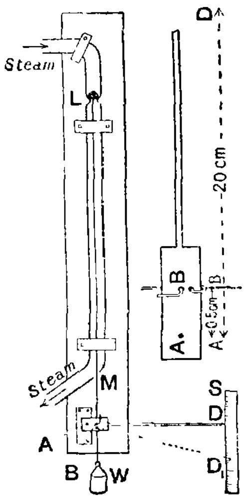


Fig. 148.

$$DD_1 = \text{(say) } 5\cdot 1 \text{ cm.}$$

Measure $LM = \text{(say) } 75$ cm.

Since $AB = 0\cdot 5$ cm. and $AD = 20$ cm.,

$$\frac{AB}{AD} = \frac{\cdot 5}{20} = \frac{1}{40},$$

$$\therefore \text{ if } D \text{ drops } 5\cdot 1 \text{ cm., } B \text{ drops } \frac{1\times 5\cdot 1}{40} \text{ cm.,}$$

but *drop* of B = expansion of the wire LM.

[1] N.B. A **metre ruler** may be substituted for the lever described above and gives satisfactory results. The two holes corresponding to A and B are drilled at 5 cm. and 10 cm. respectively. A fine nail is driven through A; the wire is hooked on to a wire loop through B.

Then $$\frac{AB}{AD} = \frac{5 \text{ cm.}}{95 \text{ cm.}} = \frac{1}{19}.$$

No weight is required.

∴ 75 cm. of Aluminium on being heated 100° – 15° C. (= 85° C.)

increases $\frac{1}{40} \times 5{\cdot}1$ cm.,

∴ 1 cm. Aluminium on being heated 1° C. increases

$$\frac{1 \times 5{\cdot}1}{40 \times 85 \times 75} = {\cdot}00002 \text{ cm.}$$

To find the Coefficient of Expansion of a solid (say *glass*) (*screw-gauge* method).

***Exp.** ii. Fig. 149 represents the apparatus. A long *glass* tube *AB* is clamped at *B* in a sloping position by a retort stand fastened securely to the table. The tube is supported at *C*. Wind a stout piece of copper wire tightly

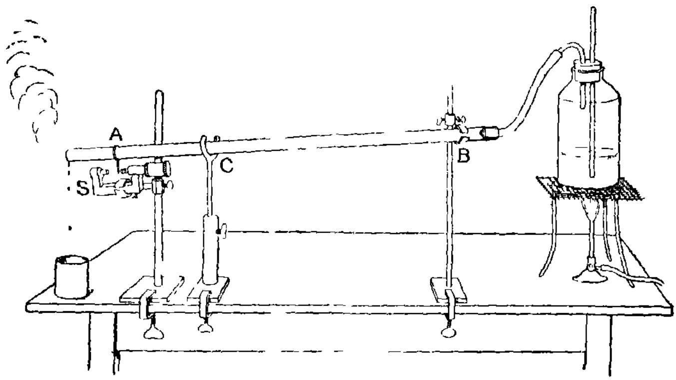


Fig. 149.

three times round the tube at *A* and twist the ends so that they form a *rigid* pointer. Measure the length of the tube from the wire *A* to the clamp *B* (= say **125·5 cm.**). Note the temp. of room (**17° C.**). Clamp a screw gauge *S* tightly to another retort stand also fastened firmly to the table. Adjust the screw gauge so that the end of the screw just touches the wire, the screw itself being fixed *parallel* to the tube. Note the reading (say **0·94 cm.**). Pass steam at **100° C.** through the tube. After 5 minutes turn the screw *carefully* so that its end just touches the wire again. Note the second reading of the screw (say **0·85 cm.**)[1].

The expansion of 125·5 cm. of glass on being heated through 100° – 17° (= 83° C.) is therefore 0·94 – 0·85 = ·09 cm.

[1] The results given in this and in several other experiments were obtained by the Fourth Forms of the Liverpool Collegiate School (aet. 14).

$\therefore$ 125·5 cm. of glass heated through 83° C. expanded ·09 cm.

$\therefore$ 1 cm. of glass „ „ 1° C. „ $\frac{\cdot 09}{125 \cdot 5 \times 83}$ cm.

$\therefore$ the coefficient of expansion of glass $= \frac{\cdot 09}{125 \cdot 5 \times 83} = \cdot 0000086$.

111. Compensated Pendulums.

The time-keeping qualities of an ordinary pendulum clock depend to a great extent on the length of the pendulum. The distance from the point of suspension to the centre of gravity of the pendulum must be constant; it must not vary with the temperature.

The Mercurial pendulum. The rod is generally made of iron[1]. The bob is a narrow cup fixed to the rod and contains mercury. The bottom of the cup moves slightly away from or approaches the point of suspension as the rod expands or contracts with variations of temperature. The mercury in the cup also expands and contracts, thus raising and lowering its centre of gravity; the quantity of mercury in the cup is so arranged that the expansion of the mercury, which is relatively greater than that of the rod, raises the C.G. as much as the expansion of the rod lowers it.

Fig. 150.

The Gridiron pendulum is made of rods of two metals having different coefficients of expansion, arranged symmetrically and fixed together "in series" at their ends (Fig. 150). Aluminium (A), coefficient of expansion, $a_A = \cdot 000022$, and Steel (S, S_1), coefficient of expansion, $a_S = \cdot 000011$, are suitable metals, their coefficients of expansion being in the proportion 2 : 1. The rods arranged as in Fig. 150, in such a way that total expansion *downwards* of the iron = total expansion *upwards* of the aluminium,

[1] "*Invar*," an alloy of steel and nickel (36 %), which retains an almost *invariable* length at different temperatures, now renders the compensated pendulums unnecessary.

i.e. $$(\text{length } S+S_1)\,\alpha_S = \text{length of } A \times \alpha_A,$$

$$\therefore \frac{\text{length of } S+S_1}{\text{length of } A} = \frac{\alpha_A}{\alpha_S} = \frac{2}{1} \text{ (see Fig. 150).}$$

EXAMPLES XVIII (EXPANSION OF SOLIDS).

1. **An iron rod measures 200 cm at 0° C and 200·12 cm at 50° C. Calculate the linear coefficient of expansion of iron.**

2. If the iron rails on a railway line are 10 yds. long what space must be left between two rails to allow for expansion if the temperature may range over 50° C.? (Lin. Coeff of Exp. of Iron = ·000012.)

3. A bar of iron is 60 ft. long at 0° C. At what temperature will it measure 60·144 ft.?

4. A zinc rod measures 100·255 cm. at 85° C. If the linear coefficient of expansion of zinc is ·00003, what will the length be at 0° C.?

5. A glass rod packed in ice is 2 metres long as measured by a brass scale at 15° C. If the scale is correct at 0° C. what is the true length of the glass? (Lin. Coeff. of Exp. of brass = ·000019.)

6. Calculate the coefficient of expansion of iron from the following data Length of Iron pipe at 20° C. = 50 cm.; Final Temperature = 100° C.; first reading of screw gauge = 9 mm.; second reading = 8·52 mm.

7. A compensated pendulum is to be constructed of Aluminium and Iron. If 3 ft. of iron is to be used what length of Aluminium will be required? (Lin. Coeff. of Exp. of Aluminium = ·000023.)

8. A line measured by a brass scale at 50° C. is 20 cm long. If the brass scale is correct at 0° C. what is the true length of the line?

9. At 0° C. a rod of iron measures 100·2 cm. and a rod of brass 100 cm At what temperature will they have the same length?

10. Calculate the coefficient of expansion of glass from the following data: Length of glass at 10° C. = 2 metres; Final temperature = 100° C.; 1st reading of screw gauge = 10 mm.; 2nd reading = 8·45 mm.

11. A compensated pendulum is to be made of Iron and Zinc. If the length of the iron is 4 ft. what length of zinc must be taken? (Coeff. of Exp of Zinc = ·00003.)

12. A wire 100 cm. long at 15° C. is attached to a metre ruler at the mark 10 cm. The ruler is capable of revolving about a pivot at the mark 5 cm. When the wire is heated to 100° C. the end of the ruler drops 1·9 cm. What is the linear coefficient of expansion of the wire?

13. A copper plate measures 10 ft. by 5 ft. at 0° C. What will its area be at 100° C.? (Lin. Coeff. of Exp. of Copper = ·000017.)

14. A lead tank is 5 ft. long, 4 ft. wide and 3 ft. deep at 0° C. How much boiling water will it hold if the linear coefficient of expansion of lead is ·000029?

15. If the volume of a brass sphere is 40 c.c. at 0° C. and 40·228 c.c. at 100° C. what is the linear coefficient of expansion of brass?

16. A copper cylinder has a diameter of 20 cm. and height 70 cm. when measured at 15° C. What will be the volume of the cylinder at 55° C.? (Lin. Coeff. of Exp. of Copper = ·000017.)

17. A glass vessel holds 2 litres at 12° C. How much will it hold at 112° C.? (Lin. Coeff. of Exp. of Glass = ·0000086.)

18. The density of lead at 0° C. is 11·4 gm. per c.c. If the coefficient of linear expansion of lead is ·000029 calculate the density at 100° C.

CHAPTER XIX.

EXPANSION OF LIQUIDS AND GASES.

A. EXPANSION OF LIQUIDS

112. Real and Apparent Expansion. When we heated the model of a thermometer (§ 105) we noticed that the column in the tube fell at first owing to the expansion of the glass flask. Let the *first* position of the top of the column be at A (Fig. 151) when both vessel and liquid are cold. If it were possible to heat the glass without warming the liquid inside, the column would fall say from A to B, and the *volume AB* would be a measure of the expansion of the *glass*. Next imagine that the *glass* does not change in volume while the temperature of the liquid is raised, then the top of the column would rise to O and BO *would measure* the *real* (or absolute) expansion of the liquid. In practice, however, both glass and liquid rise in temperature together and when the top of the column has reached O, AO represents the *apparent* expansion of the liquid. It is evident therefore that the expansion BO = expansion AB + expansion AO, or, in words,

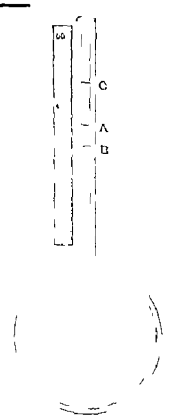


Fig. 151.

Real Expansion of Liquid

= Apparent Expansion + Expansion of the glass vessel,

or, written as coefficients of expansion (E),

$$E_{\text{Real}} = E_{\text{Apparent}} + E_{\text{Glass}}.$$

To find the apparent (or relative) coefficient of expansion of a liquid (*e.g. methylated spirits*).

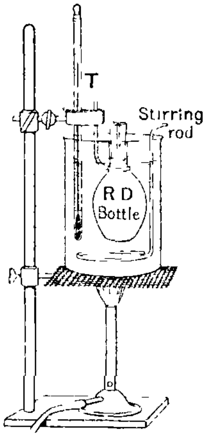


Fig. 152.

***Exp. i.** (1) Weigh an empty Relative Density Bottle (say 30·075 grams). (2) Weigh it full of methylated spirits (say 80·6 grams) and note the temperature of the laboratory (say 15·7° C.). Twist wire round the neck of the Relative Density Bottle and clamp it with a thermometer in a can of water as shown in Fig. 152. Warm the water to about 60° C., stir well, and note the temperature when constant for a few minutes (say 52·9° C.). Take out the bottle, dry it and (3) weigh again (say 78·705 grams). The loss of weight, *i.e.* (3) − (2), = **1·895 grams** = weight of spirit which has been driven out by expansion, through the liquid being heated from 15·7° to 52·9°, *i.e.* through **37·2° C.**

Let d = density of methylated spirit at 15·7° C. Then the volume at 15·7° C. of the spirit driven out $= \frac{1 \cdot 895}{d}$ c.c. The weight of spirit left in bottle after expansion = 78·705 − 30·075 = **48·63 grams**, and the volume at 15·7° C. of spirit left in bottle after expansion and which fills the bottle at 52·9° C. $= \frac{48 \cdot 63}{d}$ c.c.

$\therefore \frac{48 \cdot 63}{d}$ c.c. heated through 37·2° C. expand $\frac{1 \cdot 895}{d}$ c.c.

$\therefore$ 1 c.c. of spirit heated through 1° C. expands

$$\frac{1 \cdot 895 \times d}{d \times 48 \cdot 63 \times 37 \cdot 2} = \cdot 001047 \text{ c.c.}$$

$\therefore$ **the apparent coefficient of expansion of meth. spirit = ·001047.**

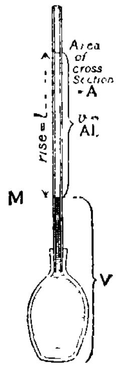


Fig. 153.

***Exp. ii.** The **coefficient of apparent expansion** may be found by *direct measurement* fairly accurately by inserting a **tube** of *known cross-section* into a small flask or bulb[1] (Fig. 153). The *area of cross-section* (A) of the tube is calculated by dividing the *vol.* by the total length. (*a*) Find

[1] Capillary tubes ground to fit Rel. Dens. Bottles or bulbs fitted with tubes may be obtained from the makers.

its *vol.* by weighing (grams) the mercury which will fill the (dry) tube and (*b*) dividing the *mass* of mercury by its *density* (13·6) (see Ex. iii, p. 75).

The volume ÷ total length of tube = area of cross-section (A).

The weighings (1) and (2) are taken as in Exp. i to obtain the *mass* (m) of liquid in bottle and tube. The *density* (d) of the liquid at the temp. of laboratory must be known. $\frac{m}{d}$ = vol. of liquid (V) at temp. of laboratory[1]. The *actual volume* of expansion may be obtained by measuring the *rise* (l) of the column in the tube, on heating through a known temperature (t) as in Exp. i and multiplying by the *area of cross section* (A) of the tube. The *rise* (l) of column in the tube is obtained by measurements from the open end.

The volume of expansion $v = A \times l$.

Then V c.c. on being heated through t° expand v,

$\therefore$ 1 c.c. on being heated through 1° expands $\frac{v}{Vt}$

= Coefficient of Expansion of Liquid.

The advantage of this *direct* method is that the coefficients of expansion of several liquids may be quickly obtained and compared when once the volume (V) and the area of cross section (A) are known.

Before heating, the level of the liquid in the tube is adjusted to a mark (M) by adding or removing liquid by a capillary tube or by a wire which acts as a plunger.

113. Effect of Expansion on Density.

We know that Density $= \frac{\text{Mass}}{\text{Volume}}$, *i.e.* $D = \frac{m}{V}$.

If D_0 and D_t represent the densities at a lower and a higher temperature respectively, and V_0 and V_t represent the corresponding volumes,

$$D_0 = \frac{m}{V_0} \text{ and } D_t = \frac{m}{V_t}, \quad \therefore \frac{D_0}{D_t} = \frac{V_t}{V_0},$$

but $V_t = V_0(1 + et)$ where e = coefficient of expansion.

$$\therefore \frac{D_0}{D_t} = \frac{V_0(1+et)}{V_0}, \quad \therefore D_0 = D_t(1 + et).$$

114. Coefficient of Absolute Expansion (Dulong and Petit's Method).

In § 45 we have learnt that the densities of two liquids are

[1] The vol. of the bulb in c.c. may be found once for all by weighing the mercury it will hold (grams) and dividing by 13·6.

inversely proportional to the heights of the columns which balance each other. $\frac{D_0}{D_t} = \frac{h_t}{h_0}$.

If then we fill a U-tube with the liquid whose coefficient of expansion we wish to determine, heat one arm of tube and cool the other, we shall have a column of *hot* liquid of greater height and less density balancing a column of *cold* liquid of less height and greater density; and *from observations of the heights of the columns we can calculate the absolute coefficient of expansion of the liquid* (e). Fig. 154 shows a U-tube containing (say **glycerine**).

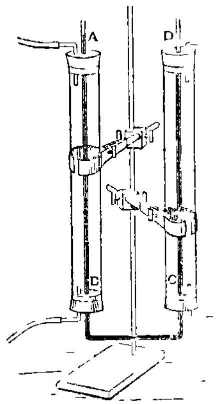


Fig. 154.

The arm CD is cooled with ice cold water at 0° C.

The arm AB is heated with steam at 100° C.

A constriction in the horizontal tube prevents currents and consequent mixing of hot and cold liquid. (1) Both sides are cooled to 0°, the length of liquid CD is measured, h_0, and the level of the cold glycerine in A is marked on the tube. Steam is then cautiously passed into the jacket on the side AB and the rise of the column is measured and added to h_0. Let the height of the hot column $= h_{100}$.

Then, since $$\frac{D_0}{D_{100}} = \frac{h_{100}}{h_0} \text{........................(§ 45)},$$

and since $$D_0 = D_{100}(1 + 100e) \text{ ....(§ 113)},$$

$$\therefore\ e = \frac{D_0 - D_{100}}{100 D_{100}} = \frac{1}{100}\left(\frac{D_0}{D_{100}} - 1\right)$$

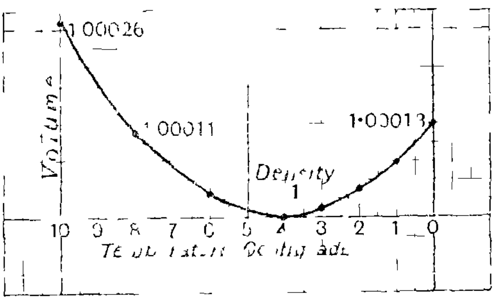
$$= \frac{1}{100}\left(\frac{h_{100}}{h_0} - 1\right) = \frac{h_{100} - h_0}{100 h_0}.$$

Coefficients of Expansion (Liquids).

Alcohol	00108	Mercury	00018
Ether	00210	Turpentine	·00105
Glycerine	00053	Water	00045

115. Peculiar expansion of water as it cools near freezing point.

Water is densest at 4° C It contracts in volume from 100° C to 4° C, but cooled below 4° C it expands and at 0° C its volume



Density at 0°
=0 99987.

Density at 4°
=1 00000

Density at 8°
=0 99989

Density at 10°
=0 99976.

Curve of Volume of Water between 10° and 0° C.

Fig. 155.

is almost exactly equal to what it was at 8° C The changes in *volume* between 10° and 0° C are shown by a curve in Fig. 155 the *density* of water is given to the right of the Fig.

Hope's Apparatus. Fig. 156 shows an apparatus which demonstrates the strange currents (§ 156) which are created in a quantity of water gradually cooled from say 10° C to the freezing point. A cylinder is filled with water at about 10° C. and two thermometers

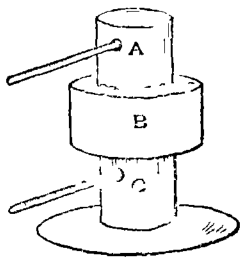


Fig. 156.

A and *C* indicate the temperatures at the top and at the bottom as the water is cooled to 0° by a freezing mixture (of ice and salt) placed in the trough *B*. The water at a medium depth near *B* is cooled, contracts in volume, becomes denser and sinks to the bottom. The thermometer at *C* shows that the water at the bottom gradually cools until 4° C. is reached. Meanwhile the water at *A* has remained at about 10° C., but now begins to show a rapid fall in temperature until it reaches 0°. This fall is due to the water near the ice jacket, being cooled below 4° C., expanding, becoming less dense and rising to the top. The thermometer at *C* remains nearly stationary at 4° C. until that at *A* has registered 0° for some minutes.

By a little imagination we can now understand the process of cooling of a pond or lake. Ice will not form until all the water of the lake has cooled to 4° C.; then the colder water being on the surface, ice will form only on the surface while the water at the bottom (if the lake is deep) will not fall much below 4° C. Fish and other aquatic animals are therefore able to exist in the deeper parts of lakes even in severe winters.

B. EXPANSION OF GASES.

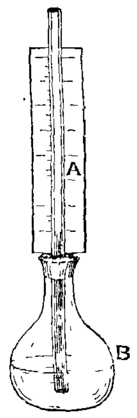


Fig. 157.

116. Exp. To show that gases expand readily on heating.

Partly fill a 500 c.c. flask *B* (Fig. 157) with coloured water. Through a rubber cork fitted to the flask put a piece of glass tube to which a paper scale *A* is attached. Arrange that the end of the tube dips below the liquid in the flask. The warmth from the hands placed round the flask is sufficient to cause a considerable expansion of the air which is shown by the rise of the column in the tube.

John Dalton an Englishman and Professor Charles a Frenchman discovered independently (about the year 1800) that equal volumes of *all gases expand almost equally when heated*, provided that the pressure is kept constant.

Coefficient of Expansion of a Gas (e).

Charles' Law. *All gases expand about $\frac{1}{273}$ of their volume at 0° C for every degree C. rise in temperature, the pressure remaining constant.*

Let V_0 and V_t be the volumes of the gas at 0° C. and at t° C. respectively. Then

$$V_t = V_0 (1 + \tfrac{1}{273} t) \qquad [\text{cf. § 109}], \qquad \tfrac{1}{273} = 00366 = (e).$$

Coefficients of Expansion of Gases.

Air	00367	Carbonic Acid Gas	00371
Hydrogen	00366	Nitrogen	00367

117. *Exp. To find the coefficient of expansion of air at constant pressure.

[*For alternative method see Appendix p 278 c.*]

Use Boyle's Law Apparatus (see § 61). A thistle funnel EK (Fig. 158) is joined to a straight open tube AQ by rubber tubing R, which should be longer than in the figure. The glass tubes are held vertically and mercury is poured into E until AQ is about half full. The *air* enclosed in the glass tube AQ must be *absolutely dry*. Two drops[1] of *pure sulphuric acid* are therefore put into AQ through the open end which is then sealed off in the blowpipe flame; the sulphuric acid absorbs moisture. The tubes are then clamped as shown in the figure. AQ is surrounded by a steam jacket. The *levels* of the mercury are adjusted on both sides, LKQ being made horizontal, and the volume V_t is measured on the scale, the temperature t° C. being noted. Steam is then passed in at 100° C from a boiler B and

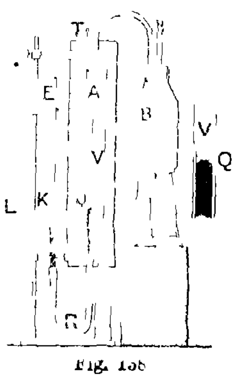


Fig. 158

[1] The quantity of sulphuric acid should be so little that the meniscus of the acid should touch the mercury (see Q at the side of Fig. 158).

the tube EK is lowered until LKQ is again horizontal; the volume V_{100} is then measured on the scale. The following are the observations taken:

$$\begin{aligned}
&\text{Position of closed end of tube } (A) \text{ on scale} = 5\cdot00 \text{ cm.}\\
&\quad,, \quad \text{mercury surface } (Q) \text{ at } 15^\circ \text{ C.} = 21\cdot73 \text{ cm.}\\
&\quad,, \quad ,, \quad ,, \quad ,, \quad 100^\circ \text{ C.} = 26\cdot66 \text{ cm.}\\
&\therefore \text{Length of column at } 15^\circ \text{ C.} = 21\cdot73 - 5 = 16\cdot73 \text{ cm.}\\
&\quad,, \quad ,, \quad 100^\circ \text{ C.} = 26\cdot66 - 5 = 21\cdot66 \text{ cm.}\\
&\qquad\qquad \text{Expansion} \qquad = 4\cdot93 \text{ cm.}
\end{aligned}$$

N.B. The area of cross section of the tube is assumed to be constant and therefore the volume of enclosed air is proportional to the length of the column.

$\therefore$ 21·66 vols. in cooling 85° would contract 4·93 vols.,

21·66 vols. ,, 1° ,, ,, $\frac{4\cdot93}{85}$ vols.,

21·66 vols. ,, 100° ,, ,, $\frac{4\cdot93 \times 100}{85} = 5\cdot8$ vols.,

$\therefore$ length of column of air (AQ) at 0° = 21·66 − 5·8 = 15·86 vols.,

$\therefore$ 15·86 vols. heated through 100° expands 5·8 vols.,

$\therefore$ 1 vol. ,, ,, 1° ,, $\frac{5\cdot8}{15\cdot86 \times 100} =$ **·00366.**

Note to Demonstrator. These tubes should be stored vertically thus, U; they may be conveniently used for Exp., § 61 (Boyle's Law), water being passed through the outer jacket to keep the temperature constant.

118. The Absolute Scale of Temperature.

We can apply Charles' Law to temperatures below 0° C., but instead of the word *expand* we must read the word *contract*. If then from unit volume we subtract $\frac{1}{273}$ of that volume for every fall of 1° C., it is evident at −273° C. we shall subtract

$$\tfrac{1}{273} \times 273 = 1,$$

therefore the volume of the gas will be reduced to nothing at −273° C.

The temperature (– 273° C.) is called the **absolute zero.** Before this temperature is reached however all gases are solidified. It is easier to understand this scale if, instead of unit volume, we consider 273 vols. at 0° C.

At	10° C.	the volume becomes	273 + 10 = 283	Corresponds with the Absolute Scale.
„	0° C.	„ „	273 + 0 = 273	
„	– 10° C.	„ „	273 – 10 = 263	
„	– 100° C.	„ „	273 – 100 = 173	
„	– 200° C.	„ „	273 – 200 = 73	
„	– 273° C.	„ „	273 – 273 = 0	

Temperature reckoned from the *absolute zero* as shown by the vols. given in the last column is called **absolute temperature.**

Degrees Centigrade + 273 = Degrees on Absolute Scale.

It is evident that *the volumes of gases at various temperatures are directly proportional to those temperatures reckoned on the absolute scale.*

Therefore if we have V_1 c.c. of air at t_1° C. and we wish to know its volume V_2 c.c. at t_2° C., we convert the temperature on the C. scale to the absolute scale by adding 273 and write:

$$\frac{V_1}{V_2} = \frac{273 + t_1}{273 + t_2} = \frac{T_1 \text{ (absolute scale)}}{T_2 \text{ (absolute scale)}}.$$

Example. 500 c.c. are measured at 15° C. What will their volume become at 100° C.?

$$\frac{500}{V_2} = \frac{273 + 15}{273 + 100}, \quad \therefore V_2 = \frac{500 \times 373}{288} = 647 \cdot 6 \text{ c.c.}$$

119. Correction of Volume of a Gas for Pressure and Temperature.

If in addition we have to correct for variation of pressure, we can imagine the process to go on in two stages: (1) allow the gases to expand or contract with variation of temperature but keep the pressure constant; (2) having obtained this result

allow the gases to expand or contract with variations of pressure but keep the temperature constant. Thus:

Example. 500 c.c. are measured at 15° C. and 700 mm. pressure. What will be their volume at 100° C. and 800 mm. pressure?

(1) Keeping the *pressure* constant

$$V = \frac{500 \times 373}{288} \qquad \text{(see Ex. above).}$$

(2) Keeping the *temperature* constant, what will be the volume of V when the pressure changes from 700 mm. to 800 mm.?

$$V_2 = \left(\frac{500 \times 373}{288}\right) \times \frac{700}{800} \qquad \text{(see § 62),}$$

hence, for *correction for Temperature and Pressure*, we obtain the formula

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2},$$

which is a combination of

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}, \text{ i.e. } \frac{V_1}{V_2} = \frac{T_1}{T_2} \text{ (see above, Charles' Law),}$$

and

$$P_1 V_1 = P_2 V_2 \qquad \text{(see § 61, Boyle's Law).}$$

For (1) $V \propto T$ when P is unaltered (Charles' Law)

and (2) $V \propto \frac{1}{P}$ „ T „ „ (Boyle's Law),

$\therefore$ $V \propto \frac{T}{P}$ „ P and T vary

or

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}.$$

120. The Coefficient of Increase of Pressure of a Gas, its volume being kept constant.

From the formula $\frac{V_1 P_1}{T_1} = \frac{V_2 P_2}{T_2}$ (§ 119).

If the volume is kept constant, *i.e.* $V_1 = V_2$, then

$$\frac{P_1}{T_1} = \frac{P_2}{T_2},$$

i.e. the pressure varies directly as the temperature reckoned on the absolute scale.

Let

$T_1 = 273°$ absolute temperature corresponding with 0° C.,

and $T_2 = (273 + t)$ „ „ „ „ t° C.

Let P_0 = pressure at 0° C.,

and P_t = " t° C.,

then $$\frac{P_0}{273} = \frac{P_t}{273 + t},$$

$$\therefore\ P_t = P_0(1 + \tfrac{1}{273}t) \qquad \text{(cf. § 116)},$$

or, in words, *the pressure of a gas increases by $\frac{1}{273}$ of its pressure at 0° C. for every rise of temperature of 1° C., the volume being kept constant* (cf. § 116).

Exp. (A) To illustrate this Law of Charles applied to Pressure use the same apparatus (Boyle's Law) as was used in Exp., § 117. Pass warm water through the jacket, Fig. 158, note the *temperature*, and raise the tube EK until the vol. V is constant, *i.e.* Q must be kept stationary, opposite to a gummed slip of paper on the water jacket. In reading the *pressures*, do not forget to add (H) the barometric pressure to the difference in level between K and Q.

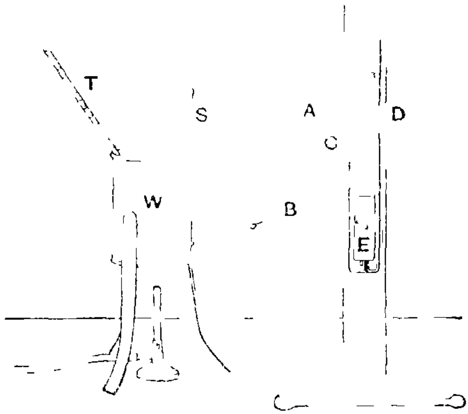


Fig. 159

(B) *Fig. 159* shows a convenient form of apparatus sometimes employed to illustrate this law. A bulb B contains *dry* air and is connected with a U-tube CD containing mercury. A reservoir consisting of a closed rubber tube full of mercury is connected with the U-tube CD. By turning a screw

E a piece of wood presses more mercury from the reservoir into CD, and as the air in the bulb tends to expand when the latter is heated in the water bath W, the thread of mercury may be constantly brought back to a point A,. by turning E; the volume of enclosed air in B is thus kept constant. By this screw device, the raising and lowering of a mercury reservoir is obviated. To the difference in level of the mercury on the two sides, the barometric pressure must be added, as mentioned above, in order to obtain the pressure of air in the bulb B. This apparatus is sometimes called an **air thermometer.**

Examples XIX a (Expansion of Liquids).

1. A graduated cylinder contains 200 c.c. of mercury at 15° C. When it is placed in a jar of water at 55° C. what volume will the mercury occupy? (App. Coeff. of Exp. of mercury = ·000155.)

2. A quantity of mercury measures 50 c.c. at 15° C. At what temperature will its volume be 50·18 c.c.? (Real Coeff. = ·00018.)

3. A density bottle holds 55 gm. of liquid at 0° C. and 50 gm. at 100° C. Calculate the coefficient of expansion of the liquid.

4. A flask weighs 20 gm. when empty and 62 gm. when full of alcohol at 0° C. What will it weigh after being heated to 50° C.? (App. Coeff. of Alcohol = ·001.)

5. 105·5 c.c. of turpentine at 50° C. become 111 c.c. at 100° C. What would the volume be at 0° C.? (Coeff. of Exp. unknown.)

6. A weight thermometer holds 200 gm. of mercury at 0° C. If heated to 100° C. how much mercury would overflow?

7. A bottle weighs 19·6 gm. when empty, 61 6 gm. when full of alcohol at 10° C. After heating to 60° C. the weight is 59·6 gm. Calculate real and apparent coefficients of expansion of the liquid.

8. A 50 c.c. bottle is filled with mercury at 0° C. What weight of mercury will be expelled when the temperature rises to 100° C.? (Density of mercury = 13·6 gm.)

9. The density of water at 80° C. = 0·972 gm. per c.c. Find the mean coefficient of expansion of water from 4° C. to 80° C.

10. A certain quantity of liquid measures 1009 c.c. at 24° C. and 1036 c.c. at 84° C. Find its volume at 4° C.

11. A flask contains 20·5 gm. of a liquid at 20° C.: on heating to 40° C. 0·5 gm. is expelled. Calculate the coefficient of expansion of the liquid.

12. How much mercury must be placed in a 50 c.c. glass bottle so that the space above the mercury may remain the same at all temperatures?

13. A weight thermometer contains 406·2 gm. of mercury at 0° C. On heating to 100° C. 6·2 gm. are expelled. If the real coefficient of expansion of mercury is ·000181, calculate the coefficient of expansion of glass.

14. The density of mercury at 0° C. is 13·6 gm. per c.c. What is its density at 100° C.?

15. A Hare's apparatus dips into two beakers one containing alcohol at 15° C., the other alcohol at 55° C. The cold column measures 50 cm.: the hot 52 cm. Calculate the coefficient of expansion of alcohol.

16. A U-tube containing aniline has one limb surrounded by ice, the other by steam. The cold column measures 50 cm.: the hot 54·5 cm. What is the coefficient of expansion of aniline?

Examples XIX B (Expansion of Gases).

Pressure constant.

1. If 192 c.c. of air measured at 15° C. become 248 c.c. at 99° C., what would the volume be at 0° C.? (Coefficient unknown.)

2. A 50 c.c. bottle full of air is heated to 100° C. while completely immersed in water. The temperature is allowed to fall to 16° C. and 12 gm. of water pass into the bottle. If the temperature fell to 0° C. how much more water would pass in? (Coefficient unknown.)

3. 300 c.c. of oxygen gas at 0° C. become 400 c.c. at 91° C. Calculate the coefficient of expansion of oxygen.

4. A quantity of air measures 91 c.c. at 0° C. What volume will it occupy at 84° C.? (Coeff. of Exp. $= \frac{1}{273}$.)

5. What rise in temperature is required to cause 150 c.c. measured at 0° C. to become 200 c.c.?

6. To what temperature must 90 c.c. of air at 0° C. be heated to treble its volume?

7. Calculate the coefficient of expansion of air from the following:
Distance of mercury surface from closed end of tube at 11° C. = 13·8 cm.: at 100° C. = 18·1 cm.

8. If at 27° C. a certain quantity of air measures 100 c.c., what will its volume be at 0° C.?

9. A quantity of oxygen measures 182 c.c. at 0° C. Calculate its volume at − 273° C.

10. A 25 c.c. relative density bottle full of air at 100° C. is inverted in a beaker of water at 16° C. 5·7 gm. of water are drawn in. Find the coefficient of expansion of air.

Boyle's Law and Charles' Law.

11. One litre of air measured at 27° C. and under a pressure of 75 cm. is cooled to 0° C. and the pressure is doubled. What is the new volume?

12. 273 c.c. of air at N.T.P. are heated to 27° C. and the pressure is halved. Calculate the new volume.

13. 91 c.c. of air at N.T.P. are heated to 60° C. and at the same time the pressure is changed. If the air finally measures 55·5 c.c. what was the pressure?

14. A certain quantity of oxygen occupies 150 c.c. at 27° C.: at 77° C. when the barometer stands at 76 cm. the volume is 210 c.c. Under what pressure was the gas originally measured?

15. A quantity of gas measured at 102° C. and under a pressure of 750 mm. occupies 76 c.c. Calculate the volume at N.T.P.

16. A litre of gas measured at − 3° C. and under a pressure of 74 cm. is heated to 24° C. while the pressure is increased to 76 cm. What is the new volume?

17. Find the volume at N.T.P. of a quantity of gas which measures 804 c.c. at 47° C. under a pressure of 80 cm.

CHAPTER XX.

(A) CALORIMETRY. (B) SPECIFIC HEAT.

121. Calorimetry is Measurement of Quantity of Heat.

In § 104 we showed experimentally that a red hot piece of iron does not necessarily contain more heat than another piece of iron which is only warm. We roughly compared the *quantities of heat* contained in two pieces by testing their "temperature raising powers" on *equal* quantities of cold water. This was an instance of the kind of method employed to *measure the quantity of heat* in a body—to try the "temperature raising power" of the body on mixing it with a *known quantity* of another substance, which is usually *water*.

Exp. To find the effect of mixing equal masses of water at different temperatures. Take two beakers A and B of the same size (150 c.c. capacity). Into one of the beakers (A) put 50 c.c. of water and warm it to about 40° C. Into the second beaker (B) measure 50 c.c. of ice cold water. Take the temperatures of each quantity exactly, and quickly pour the contents of A into B and then pour the whole quickly back again into A[1]. Note the

[1] If the temperature of the room is (say) 17°, the cold water gains and the warm water loses approximately the same quantities of heat while the experiment is proceeding. The temperature of the mixture is approximately that of the room, hence there will be little error due to loss or gain of heat after mixing. The object of pouring the mixture back again into A is (1) to thoroughly mix the two quantities, (2) that the beaker A which is still warm may restore the heat lost by the mixture in raising the temperature of beaker B.

final temperature of the mixture. It will be found that the temperature of the mixture is almost exactly midway between the temperatures of the warm and the cold water: in other words the 50 c.c. of warm water have lost as much heat as the 50 c.c. of cold water have gained.

In mixing a hot with a cold body, if there is no heat lost by radiation,

heat lost by hot body = heat gained by cold body.

122. Unit of Heat or Calorie.

Let us write the quantities of heat gained and lost by the cold and hot water in Exp., § 121, in terms of a unit which is defined as follows:

The unit of Heat is the quantity of heat given out or taken in by 1 gram of water when its temperature falls or rises 1° C. This unit is called a **Calorie** or the "**gram-degree**" unit of heat.

Observations of Exp., § 121. Heat units gained and lost.

Mass of *warm* water 50 grams [M], temperature 39° C. [$T°$].
" *cold* " 50 grams [m], " 1° C. [$t°$].
Temperature after mixing 19·8° C. [$\theta°$].

Fall in temperature of *warm* water $= 39 - 19{\cdot}8 = 19{\cdot}2°$ C.
Rise " *cold* " $= 19{\cdot}8 - 1 = 18{\cdot}8°$ C.

When 1 gram of water falls 1° C. in temp. it loses 1 calorie. [Definition.]

∴ 50 grams of water falling 19·2° C. in temp. **lose** $50 \times 19{\cdot}2$ $=$ **960 calories,**

and

50 grams of water rising 18·8° C. in temp. **gain** $50 \times 18{\cdot}8$ $=$ **940 calories.**

Note. If there were no experimental errors, heat *gained* = heat *lost*.

***Exp. To find the effect of mixing different masses of water at different temperatures.** Take 40 grams of hot water at about 50° C. and 60 grams of ice cold water. Observe the temperatures as in Exp., § 121. Pour quickly the cold water into the hot and the mixture back into the cold beaker. Stir with the thermometer and note the *final temperature*.

Observations.

Mass of *hot* water 40 grams [M] at temp. 50° C. [T].

" *cold* " 60 " [m] " 1° C [t].

Final temperature after mixing 20·5° C. [θ].

Fall of temp. of *hot* water $= T - \theta = 50 - 20{\cdot}5 = 29{\cdot}5°$ C.

Rise " *cold* " $= \theta - t = 20{\cdot}5 - 1 = 19{\cdot}5°$ C.

∴ **Heat lost** by *hot* water $= M(T - \theta) = 40 \times 29{\cdot}5 = \mathbf{1180}$ **calories.**

" **gained** by *cold* " $= m(\theta - t) = 60 \times 19{\cdot}5 = \mathbf{1170}$ **calories.**

Hence (approx.)

heat lost by *hot* water = *heat gained* by *cold* water,

and

mass of *hot* water × fall in temp. = mass of *cold* water × rise in temp.,

or $$M(T - \theta) = m(\theta - t).$$

123. Capacity for Heat. Repeat the last two experiments, but *instead of* cold *water* use cold *methylated spirit* or *turpentine*. The final temperature of the mixture is *higher* than when water was used as the cold body. Therefore less heat is required to raise one gram of methylated spirits or turpentine 1° C. than was required to raise the same weight of water 1° C. Their **capacity for heat** is less than that of water.

Exp. To show that different metals have different capacities for heat. Small spheres of *equal weight* of *Aluminium, Iron, Copper* and *Lead* are suspended from a ring by fine wire and heated to 100° C. in boiling water. A thick sheet of wax is placed ready at hand. The metal spheres are quickly taken from the boiling water and placed on the wax. Their power of melting wax depends on their capacities for heat. Those named first sink greater distances into the wax than those occurring later in the list, showing that different metals vary in their capacities for heat.

Definition of Specific Heat.

The **specific heat** of a substance is *the ratio of the quantity of heat required to raise the temperature of a given mass of a substance through* 1° *C. to the quantity of heat required to raise an equal mass of water through* 1° *C.*

Specific Heat

$$= \frac{\text{No. of heat units required to raise } m \text{ grams of subst. } 1^\circ \text{ C.}}{\text{No. of heat units required to raise } m \text{ grams of water } 1^\circ \text{ C.}}$$

= No. of calories required to raise 1 gram of the substance 1° C.

Therefore, if S = Specific Heat of a substance,

i.e. S = no. of calories required to raise 1 gram of subst. 1° C., then

$S \times m \times t$ = no. of calories required to raise m grams of subst. t° C.

124. The **Calorimeter.**

Fig. 160 shows the apparatus in which observations in heat-

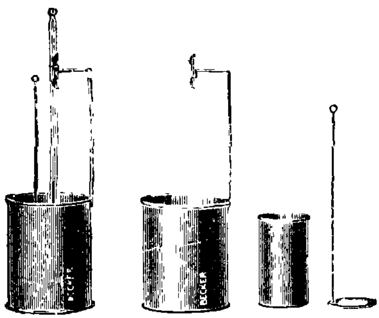


Fig. 160.

measurement are generally made. A small very light copper vessel called the *calorimeter* is placed on a perforated cork (non-conductor of heat) in a larger protecting vessel. A stand to which an accurate thermometer may be fixed with a rubber band and a stirrer complete the apparatus. For method of using see § 125.

The **water equivalent of the calorimeter** (W. E.) is the mass of water which has the same capacity for heat as the calorimeter. Both water and calorimeter receive heat from the hot body and therefore the W. E. must be added to the mass of water in the calorimeter[1].

[1] It is usual to weigh the copper *stirrer* with the calorimeter, of which it is considered a part.

***Exp. To find the Water Equivalent of the Calorimeter (W.E.).**

(i) *By calculation.*

If μ is the mass of the Calorimeter,

and σ „ Specific Heat of its material,

then $$\text{W.E.} = \mu\sigma.$$

(ii) *By experiment.*

(1) Weigh the calorimeter .... 20 grams.
Pour about 50 grams of cold water into the calorimeter.

(2) Weigh the calorimeter and cold water .... 70 grams.

(3) Note the temperature .... 15° C.

(4) Pour in about 30 grams of hot water at .... 42·3° C.

(5) Note the temperature of the mixture .... 25° C.

(6) Weigh the calorimeter + mixture .... 100 grams.

Then

heat given out by the hot water = Wt. × Fall in temp. = 30 × 17·3 = 519 calories,

„ *taken in* „ *cold* „ = Wt. × Rise in temp. = 50 × 10 = 500 calories.

∴ Heat taken in by Calorimeter = 519 − 500 = 19 calories,

∴ 19 calories raise the temp. of Calorimeter 10° C.,

∴ 1·9 „ „ „ „ 1° C.,

i.e. 1·9 grams of water would be raised 1° C. by the calories which raise the temp. of the calorimeter 1° C.

∴ **W.E. = 1·9 grams.**

*125. Exp. To find the S.H. of Marble Chips by the method of mixtures.

Observations.

(1) Weigh some marble chips .... 30 grams.

(2) Weigh the calorimeter and stirrer (copper S.H. = ·095) (μ) 25 grams.

(3) Weigh the calorimeter and cold water .... 105 grams.

∴ Weight of water (m) .... 80 grams.

Place the marble in a boiling tube and insert a thermometer; plug up the end of the tube with cotton wool and heat the tube in a beaker of boiling water until

(4) The temperature (T) of the marble is steady .... 98° C.

(5) Observe the temperature (t) of the cold water in calorimeter...15° C.

Quickly take the boiling tube from the hot water and remove the plug of cotton wool, allowing the marble to drop quietly into the calorimeter. Take care that no condensed steam drops into the calorimeter. Stir quickly and

(6) Note the final temperature θ ....21·2° C.

Then

Heat *lost* by Marble = Mass of Marble × its S.H. × fall in temperature,

and

Heat *gained* by Water + heat *gained* by Calorimeter = Mass of Water × rise in temperature + Mass of Calorimeter × its S.H. × rise in temperature,

or

$$M \times \text{S.H.} \times (T - \theta) = (m + \overline{\mu \times {\cdot}095})(\theta - t),$$

$$\therefore\ 30 \times \text{S.H.} \times (98^\circ - 21{\cdot}2^\circ) = (80 + \overline{25 \times {\cdot}095})(21{\cdot}2^\circ - 15^\circ),$$

$$\therefore\ \text{S.H. of Marble} = \frac{(80 + \overline{25 \times {\cdot}095})(21{\cdot}2 - 15)}{30 \times (98 - 21{\cdot}2)} = \mathbf{0{\cdot}22}\ \text{Calorie}$$

*126. To find the S.H. of Turpentine (assuming the S.H. of Copper to be ·095) by the Method of Mixtures.

Let S.H. = Specific Heat of Turpentine.

Observations.

(1) Weigh some dry *copper rivets* (S.H. $\cdot 095 = \sigma$) (M) ....100 grams.

(2) Weigh a calorimeter and stirrer (copper S.H. $\cdot 095$) $= \mu$ ......25 grams.

∴ Water Equivalent of Calorimeter (W) = ·095 × 25 = 2·375 grams.

(3) Weigh calorimeter + stirrer + cold turps. ....105 grams.

∴ Weight of turps. (m) ....80 grams.

Put the rivets in a boiling tube and insert a thermometer and plug of cotton wool, placing the tube in a beaker of boiling water until the

(4) Temperature (T) of the rivets is steady ....98° C.

(5) Observe the temp. (t) of the cold turps. in Calorimeter ....15° C.

Pour the rivets quickly into the turps. in the Calorimeter,

(6) Stir well and note the final temperature (θ) ....31° C.

Then

Heat *lost* by Rivets $= M \times \sigma \times (T - \theta) =$ Mass of Rivets $\times$ S.H. $\times$ fall,

and

$$\left.\begin{array}{l}\text{Heat } gained \text{ by Turps.} \\ +\text{ heat } gained \text{ by} \\ \text{Calorimeter}\end{array}\right\} = \left\{\begin{array}{l} m \times \textbf{S.H.} \times (\theta - t) \\ \quad + W \times (\theta - t)\end{array}\right\} = \left\{\begin{array}{l}\text{Mass of Turps} \times \textbf{S.H.} \times \text{rise} \\ + \text{Water Equivalent} \times \text{rise}\end{array}\right.$$

$$\therefore 100 \times \cdot 095 \times (98 - 31) = 80 \times \textbf{S.H.} \times (31 - 15) + 2 \cdot 375 \times (31 - 15),$$

$$\therefore \textbf{S.H.} \text{ of Turpentine} = \frac{100 \times \cdot 095 \times 67 - 2 \cdot 375 \times 16}{80 \times 16} = \mathbf{0 \cdot 47} \text{ Calorie.}$$

*127. To find the temperature of the Bunsen Flame by heating a *brass* ball in it and immersing in water.

Here T is unknown. Proceed as before except that the stirrer may be dispensed with. Suspend the *brass* ball, which must be previously weighed, by a thin brass wire in the Bunsen flame for five minutes; then lower it quickly into the known weight of water in the calorimeter. Observe M, m, W, t and θ as in the previous Exp. and assume that S. H. of brass $= \cdot 094$ (Calorie), then all values are known except T in the equation

$$\cdot 094\, M (T - \theta) = (m + W)(\theta - t).$$

*128. To find the S.H. of a Liquid (say Glycerine) by the Method of Cooling.

(1) Weigh a calorimeter (20 grams). Put 50 c.c. of glycerine into a beaker and heat it up to 55° C. Pour the glycerine into the calorimeter and notice the *time* taken in cooling from 50° to 40° C (say) $T_G = \mathbf{196}$ **secs.**

(2) Weigh calorimeter + glycerine $= 83$ grams.

$\therefore$ Wt. of 50 c.c. of glycerine $= 63$ grams.

Pour the glycerine into the store bottle, dry the calorimeter and place in it 50 c.c. of water at 55° C. Note the *time* it takes to cool from 50° to 40° C. (say) $T_W = \mathbf{420}$ **secs.**

The conditions of cooling should be the same for both liquids.

Let S. H. = sp. ht. of glycerine.

Then

heat given out by the glycerine per sec.

$$= \frac{\text{Mass} \times \text{fall in temp.} \times \text{S. H.}}{\text{Time}_{\text{Glycerine}}} \text{ Calories,}$$

$$\text{heat given out by the water per sec.} = \frac{\text{Mass}_W \times \text{fall in temp}}{\text{Time}_{\text{Water}}} \text{ Calories,}$$

and since the surfaces exposed are equal in area for both and the conditions are the same, the quantities of heat given out per sec. are equal.

$$\therefore \frac{63 \times 10 \times \text{S.H.}}{196} = \frac{50 \times 10}{420},$$

$$\therefore \text{S.H.}_{\text{Glycerine}} = 0{\cdot}37 \text{ Caloria.}$$

Table of Specific Heats.

Alcohol	·6	Glass	·16	Marble	·22
Aluminium	·21	Glycerine	·37	Mercury	·033
Brass	·094	Ice	·5	Turpentine	·47
Copper	·095	Iron	·114	Water	1·00

129. The high Specific Heat of the Oceans affects climate.

From the above table we notice that water has a much higher specific heat than any other substance. The capacity for heat of the oceans being so great, these large volumes of water take a long time to cool down if warm, or to be raised in temperature if cold. These facts have great influence on the climate of countries with an extended sea-board. The sea then is a vast receptacle for heat and tends to moderate the extremes of climate. The ocean currents from the tropics and from the polar regions, traversing wide ranges of the globe, cause variations of the climate for places of the same latitude.

EXAMPLES XX A (CALORIMETRY).

1. Half a kilogram of water is heated from freezing point to boiling point. How many calories are required?

2. What weight of water would be heated 6·4° C. by 800 calories?

3. How many calories are required to raise the temperature of 80 gm. of water from 15° C. to 95° F.?

4. Calculate the loss of heat in the following experiments when hot and cold water is mixed:

		(a)	(b)
Weight of calorimeter empty		= 40 gm.	22 gm.
,, ,, and cold water		= 90 gm.	72 gm.
,, ,, ,, mixed hot and cold water		= 117 gm.	130 gm.
Temperature of cold water		= 15° C.	15° C.
,, hot ,,		= 30° C.	34° C.
,, mixture		= 20° C.	25° C.

5. 60 gm. of water at 45·5° C. are mixed with 40 gm. of water at 12° C. Find temperature of mixture.

6. If 40 lb. of hot water are poured into 50 lb. of water at 10° C. and the final temperature is 30° C., what was the temperature of the hot water?

7. How much water at 104° F. will be required to raise the temperature of 100 gm. of water from 13° C. to 25° C.?

8. A can at 14° C. is filled with 100 gm. of water at 56° C. After stirring, the temperature is 54° C. How much heat has passed into the can? How much heat would raise the can's temperature 1° C.? How many grams of water would be raised 1° C. by the same amount of heat?

9. 50 gm. of water at 59° F. are mixed with 50 gm. of water at 59° C. What is the temperature of the mixture?

10. 40 gm. of water at 45° C. are poured into a calorimeter containing 78 gm. of water at 15° C. The temperature of the mixture is 25° C. How much heat has passed into the calorimeter? What is its water-equivalent?

11. A calorimeter of water-equivalent 3·5 gm. contains 41·5 gm. of water at 15° C.: 25 gm. of water at 50° C. are poured in. What is the final temperature?

12. A calorimeter of water-equivalent 4 gm. contains 50 gm. of water at 15° C. How much water at 34° C. must be poured in to raise the temperature to 25° C.?

Examples XX b (Specific Heat).

1. (*a*) How many calories are needed to raise the temperature of 100 gm. of copper from 15° C. to 95° C.? (*b*) What rise in temperature is caused if 50 gm. of copper receive 95 calories? (*c*) What weight of copper can be raised from 15° C. to 65° C. by 380 calories?

2. 20 gm. of copper at 100° C. are dropped into 30·4 gm. of water at 15° C. If the final temperature is 20° C. calculate the S. H. of copper.

3. 100 gm. of mercury at 100° C. are poured into water at 16° C. and the final temperature is 20° C. What was the weight of the water?

4. What will the final temperature be if 50 gm. of tin (S. H. = ·056) at 100° C. are mixed with 56 gm. of water at 16° C.?

5. 500 gm. of copper are heated in a furnace and dropped into 1000 gm. of water at 15° C. If the mixture rises to 53° C. what was the temperature of the furnace?

6. 250 gm. of lead (S. H. = ·031) at 100° C. are poured into a calorimeter containing 120 gm. of water at 15° C. If the final temperature is 20° C. calculate the water-equivalent of the calorimeter.

7. If 40 gm. of copper at 10° C. are mixed with 40 gm. of turpentine (S. H. = ·5) at 50° C. what is the final temperature?

8. A glass beaker weighing 52 gm. contains 60 gm. of water at 10° C. Into it 90·5 gm. of glass beads at 98° C. are poured and the final temperature is 28° C. Calculate the S. H. of glass.

9. A calorimeter of water-equivalent 3 gm. contains 52 gm. of water at 14° C. Into it 37·5 gm. of metal at 97° C. are poured. If the final temperature is 17° C. what is the specific heat of the metal?

10. A charcoal button weighing 1 gm. is heated in the electric arc and when dropped into 50 gm. of water at 15° C. causes the temperature to rise to 28° C. If the S. H. of charcoal is ·19 what is the temperature of the arc?

11. 50 gm. of lead (S. H. = ·031) at 97° C. are poured into 76 gm. of alcohol at 14·5° C. contained in a calorimeter of water-equivalent 4 gm. The final temperature is 17° C. Calculate the specific heat of alcohol.

12. Copper rivets weighing 24 gm. at 99° C. are poured into a copper calorimeter (weight = 20 gm.) containing 39·5 gm. of water at 4° C. If the temperature of the mixture is 9° C. what is the S. H. of copper?

CHAPTER XXI.

CHANGE OF STATE. FUSION OR MELTING.

130. Melting point. Let us call to mind what happened when we determined the *freezing point* on the stem of a thermometer (§ 106). We surrounded the bulb with *melting ice* and allowed the water to drain away. Heat was being added to the ice causing it to melt and yet the temperature of the ice was not raised above 0° C. This change of state of a pure substance from solid to liquid takes place at a definite temperature which is called the **melting point**, so that many substances may be recognized by determining their melting points.

***Exp. To find the Melting Point (M.P.) of Paraffin Wax** (Tube Method). Draw out a piece of glass tubing into a capillary tube[1]. Put a little wax into the wider part of the tube, gently warm it so that it runs into the capillary tube, seal the end and attach the tube C to the stem of the thermometer T, as shown in Fig. 161, by rubber bands. The paraffin in P should be opposite the bulb B. Hang the thermometer on a retort stand; put water and a stirrer S into the beaker and support the beaker by wire gauze on a ring of the retort stand. Heat the water gently, stirring at the same time. Find the melting point approximately; then allow the water to cool, stirring gently, and notice the temperature (t_1°) *accurately* when a cloudiness first appears in P. Allow all the paraffin to solidify, then warm slowly again and accurately note the temperature (t_2°) when the edge of the paraffin appears clear. Note that $t_2° > t_1°$. The mean of t_1° and t_2° is the **melting point**.

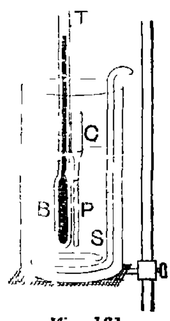


Fig. 161.

[1] See *Experimental Science*, II, Chemistry, § 3.

131. *Exp. To find the temperature of solidification of a solid by a curve of cooling.

About 20 grams of the substance are placed in a boiling tube which is clamped in position in the middle of a beaker containing a suitable liquid boiling at a considerably higher temperature than the M.P. of the solid; *e.g.* if the solid is *paraffin wax* (M. P. 55° C.), *bees' wax* (M. P. 62° C.), or *naphthalene* (M. P. 79° C.) the surrounding liquid in the beaker should be water if the M. P. of *sulphur*[1] (M. P. 114° C.) were to be determined, the "bath" may be sulphuric acid, but *great care* is required if the latter is used. A thermometer is placed in the melted substance, the source of heat is removed and the temperature noted every minute, until solidification is complete. A curve— temperature/time— is plotted (§ 13) which will show that at one period during the cooling the temperature remains constant; this *temperature* is the *solidifying point* required.

132. The Latent Heat of Fusion.

We must next enquire more carefully into the quantity of heat required to melt ice without raising its temperature. What becomes of the heat? How much heat is required to melt a gram of ice? The word "latent" means "hidden." The heat that is absorbed by *one gram* of a body on melting without producing rise of temperature is called the **latent heat of fusion,** which in the case of *ice* is *the number of calories required to convert one gram of ice at 0° C. to water at 0° C.*

***Exp. To find the Latent Heat of Fusion of Ice** ($=L$).

Weigh a calorimeter and stirrer .... **76·36** grams.

Half fill it with warm water at about 25° C. and weigh calorimeter, stirrer and water ....116·44 grams.

∴ Wt. of **warm water** (M) = 116·44 − 76·36 = **40·08** grams.

Note the temperature of the room ....15° C.

and the temperature of the water ($T°$) ....21·8° C.

Obtain some small pieces of ice, dry them on blotting paper and drop them with blotting paper "tongs" into the water until the temperature is as many degrees below the temperature of the room as the warm water was above it. Stir well until the ice is melted and note the temperature of the mixture (θ) .... 9° C.

[1] Sulphur solidifies at 120° C.

Temperature of ice, t° ....0° C.

$\therefore$ *Rise* in temp. of "Ice Water" $= \theta^\circ$ ....9° C.

Weigh the calorimeter, stirrer, water + "Ice Water" ....123·14 grams.

$\therefore$ Wt. of ice $(m) = 123\cdot14 - 116\cdot44 =$ ....**6·7** grams.

Fall in temp. of warm water $= T^\circ - \theta^\circ = 21\cdot8^\circ - 9^\circ =$ **12·8° C.**

Then

$$\left.\begin{array}{l}\text{Heat } \textit{taken in} \text{ by Ice in melting and} \\ \text{by "Ice Water" in rising in temp.}\end{array}\right\} = \left\{\begin{array}{r}\text{Heat } \textit{given out} \text{ by Warm Water} \\ + \text{Calorimeter (copper),}\end{array}\right.$$

$$mL + m\theta = M(T - \theta) + \mu\sigma(T - \theta),$$

$$6\cdot7L + 6\cdot7 \times 9 = 40\cdot08 \times 12\cdot8 + (76\cdot36 \times \cdot095 \times 12\cdot8),$$

hence by calculation $\qquad L = $ **81·43** calories,

more accurately

the Latent Heat of Fusion of Ice = 79·67 calories.

The fact that a considerable amount of heat is required to melt ice and also that from ice-cold water much heat must be taken to convert it to ice, in nature causes ice to form slowly and melt slowly. If L were small, rivers, lakes and seas would be rapidly changing from ice to water and water to ice with consequent disastrous effects. It is easy now to understand why the presence of melting icebergs in the North Atlantic causes such cold winds and climate in the British Isles.

133. Heat absorbed on dissolving certain crystals.

When Ammonium Chloride (Sal Ammoniac), Ammonium Nitrate and some other substances are dissolved in water the temperature of the solution falls. Heat energy is absorbed when the crystals pass into the liquid state.

***Exp. To find (*a*) the proportions in which Sal Ammoniac dissolves in water in producing the greatest absorption of heat, and (*b*) to calculate approximately the heat units absorbed.**

Into a weighed calorimeter put about 50 grams of water at the temperature of the room. Find the weight of the water. Gradually stir in with a thermometer finely ground crystals of Sal Ammoniac, and note the lowest temperature obtained when all the crystals are dissolved. Weigh the whole to find the weight of crystals added.

Assume that the specific heat of Sal Ammoniac in solution is the same as that of water, and calculate the number of calories absorbed per gram of crystals dissolved.

134. Freezing Mixtures.

Mixtures of Salt and Ice, of Sal Ammoniac and Ice, and of Ammonium Nitrate and Ice are *freezing mixtures.* The salts mentioned are very soluble in water—heat energy is absorbed when they pass into solution and also when the ice melts, consequently any other body near gives up its heat to effect these changes with a consequent reduction of its temperature.

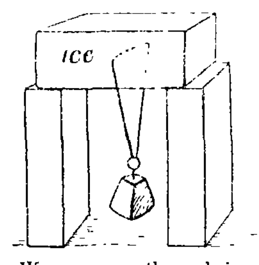
***Exp.** Try the conditions and proportions most suitable to obtain the greatest reduction of temperature with any of the above freezing mixtures.

Exp. To show that heat is given out on solidification.

The reverse process to that of § 133 may be shown by melting crystals of "hypo" (hyposulphite of soda) in a small flask, plugging the mouth with cotton-wool to prevent the access of dust, and setting the flask aside to cool. When it is quite cold on inserting a thermometer and a small crystal of "hypo," the whole solution crystallizes out and the thermometer shows a *rise* in temperature on solidification.

135. Increase of Volume when Water freezes.

1 c.c. of water at 0° becomes 1·091 c.c. of ice at 0°. Great pressure is exerted by the ice if the change takes place in a confined space; water pipes and strong iron vessels are burst open. **Pressure** however **lowers the freezing point** but only to a slight extent. This is evident when we remember how snow binds together and pieces of ice stick together when pressed provided their temperature is not much below 0°. A stone placed on ice gradually sinks into it and a piece of wire, to which a heavy weight is attached, as in Fig. 162, will gradually cut through the ice and yet leave it intact. Under the wire ice melts at increased pressure, above the wire it



Wire passing through ice.
Fig. 162.

forms again as the pressure is reduced, consequently the wire travels through the ice in the direction of pressure.

136. Ice Calorimeter.

Black's calorimeter is the simplest (Fig. 163 *a*). A smooth hole is bored in a block of ice and a slab of ice is placed to cover the hole which is wiped dry with a sponge. Suppose that we wish to find the specific heat of a piece of *iron*. The piece of iron is weighed ...**50** grams.

It is heated to 100° C. in a steam heater and quickly placed in the hole and covered by the slab. As the iron cools ice is melted.

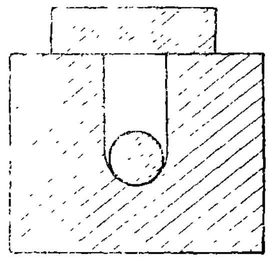


Fig. 163 *a*.

A dry sponge is weighed ....18 grams.

The iron and the hole are wiped dry by the sponge and the latter is weighed again.......................................25·2 grams.

∴ Weight of ice melted = ... 7·2 grams.

If the latent heat of fusion of ice = ...79·5 calories,

Heat lost by the iron in cooling through 100° = 7·2 × 79·5 calories.

∴ 50 grams of iron in cooling 100° C. lost 7·2 × 79·5 calories,

$$\therefore \quad 1 \text{ gram} \quad ,, \qquad ,, \quad 1^\circ \text{ C.} \quad ,, \quad \frac{7{\cdot}2 \times 79{\cdot}5}{50 \times 100}$$

$$= 0{\cdot}114 \text{ calories.}$$

∴ Specific Heat of Iron = 0·114.

Lavoisier and Laplace's ice calorimeter (Fig. 163 *b*) measures the quantity of heat in a body by measuring the weight of ice at 0° C. which is melted by the body. A small vessel *A* for the reception of the hot body is surrounded by a jacket *B* containing ice, which is itself protected from radiation and air currents by a second jacket *C* containing ice. On introducing the hot

body into the inner vessel ice melts in the middle chamber B and the water formed is collected and weighed (m).

The number of Heat Units gained by the cold body $= mL$

$=$ Weight of Ice melted (grams) $\times$ Latent Heat of Fusion of Ice.

If M = mass of hot body, S = its Specific Heat and t° C. = its temp. when placed in the calorimeter, then

the number of Heat Units *lost* by the *hot* body $= MSt$

$=$ the number of Heat Units *gained* by the *cold* body $= mL$,

$$\therefore\ S = \frac{mL}{Mt}.$$

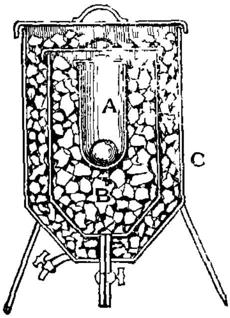


Fig. 163 *b*.

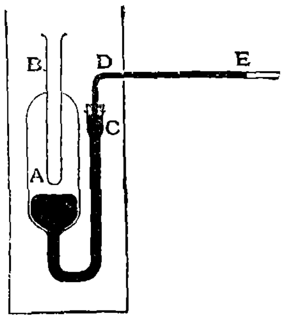


Fig. 164.

Bunsen's Ice Calorimeter. Use is made of the fact that there is a change in volume when ice melts. The hot body is introduced into a tube B (Fig. 164) surrounded by solid ice and water A contained in a closed vessel which is connected with a gauge DE which shows change of volume. Knowing that 1·09 c.c. of ice become 1 c.c. of water, we can calculate the weight of ice melted (m) from the change in volume registered by the gauge. Then

$$mL = \text{calories lost by the hot body}$$
$$= MSt,$$
$$\therefore\ S = \frac{mL}{Mt} \text{ as above.}$$

For details see Appendix, p. 258 *b*.

EXAMPLES XXI (LATENT HEAT OF FUSION).

1. How many grams of ice can be melted by 400 calories? (Latent heat of fusion = 80.)

2. How many calories would be needed to melt 20 gm. of ice and raise the temperature of the melted ice to 15° C.?

3. If the specific heat of ice is 0·5, how many calories will be required to change 10 gm. of ice at − 20° C. to water at 20° C.?

4. How many grams of boiling water will be needed to just melt 1 kgm. of ice?

5. If 2 gm. of ice receive 200 calories, what will the final temperature be?

6. How much ice at 0° C. must be put into 60 gm. of water at 40° C. to reduce the temperature to 0° C.?

7. What weight of ice at 0° C. must be added to 50 gms. of water at 100° C. to reduce the temperature to 20° C.?

8. If 5 gm. of ice at 0° C. are added to 30 gm. of water at 50° C., what is the final temperature?

9. 5 gm. of ice at 0° C. are dropped into 21·25 gm. of water at 25° C. and the final temperature is 5° C. What is the latent heat of fusion of ice?

10. 20 gm. of ice at 0° C. are mixed with 40 gm. of boiling water and the temperature is 40° C. when all the ice has melted. Calculate the latent heat of water.

11. A calorimeter, whose water-equivalent is 4 gm., contains 51 gm. of water at 24° C. How much ice must be added to reduce the temperature to 8° C. when it has all melted?

12. Calculate the latent heat of water from the following data:— weight of copper calorimeter = 40 gm.: calorimeter and warm water = 90 gm.: calorimeter, water and ice = 96 gm.: temperature of warm water = 20° C.: final temperature = 10° C. (S. H. of copper = 0·1.)

13. 80 gm. of copper (S. H. ·095) are heated to 100° C. and then placed in a cavity in a block of ice. How much ice will be melted?

14. How much heat is required to raise 100 gm. of tin from 12° C. to its melting-point, 232° C., and to melt it. (L. H. of fusion = 14·25.)

15. If 8 gm. of ice are added to 50 gm. of boiling water, what will be the resulting temperature?

16. If 5 gm. of ice melt, what change in volume takes place?

17. Calculate the latent heat of fusion of ice from the following:—weight of copper calorimeter = 50 gm.: calorimeter and water = 72 gm.: calorimeter, water and ice = 75 gm.: original temperature = 20° C.: final temperature = 10° C. (S. H. of copper = 0·1.)

18. A platinum ball weighing 40 gm. (S. H. ·03) is heated in a Bunsen flame and then placed in an ice-calorimeter. If 15 gm. of ice are melted, what was the temperature of the flame?

19. How much heat is required to raise 50 gm. of zinc from 22° C. to its melting point, 422° C., and to melt it? (S. H. of zinc = ·096: latent heat of fusion = 28·13.)

20. What will be the final temperature if 10 gm. of ice at −4° C. are mixed with 90 gm. of water at 30° C.?

21. How much heat must be passed into a mixture of ice and water to cause a decrease in volume of 1 c.c.?

CHAPTER XXII.

CHANGE OF STATE. VAPORIZATION.

137. Evaporation and Boiling.

We are familiar with the fact that liquids exposed to air evaporate: ponds and puddles dry up, muddy roads become hard and dusty, wet clothes are dried in the wind and sun or before the fire. We notice that evaporation is aided (1) when heat is supplied, (2) when the process takes place in a draught. Let us investigate the conditions and try to find whether there is any analogy (*a*) between evaporation and boiling, (*b*) between the processes and the change from solid to liquid where an absorption of heat energy occurred.

***Exp. i.** Weigh an evaporating dish about half-full of water (w). (*a*) Warm it to 50° C. on a sand bath and let it remain at this temperature for 10 minutes and weigh again (w_1): (*b*) continue to keep it at 50° C. but keep a constant draught across the surface by blowing for 10 minutes; weigh again (w_2): (*c*) increase the heat and keeping the temperature at 80° for 10 minutes under the same conditions as (*b*) weigh again (w_3). It will be found that $(w_2 - w_3) > (w_1 - w_2) > (w - w_1)$. It is unnecessary to raise the water to the boiling point (100° C.) and repeat the weighings, because experience tells us the evaporation is greatly aided by boiling.

***Exp. ii.** Transfer the hot water to a flask or beaker and boil it. Notice that bubbles of gas appear on the hot sides of the vessel which at first condense as they pass through the cooler layers of liquid but later they rise to the surface and pass off as vapour. These bubbles are *gaseous* water, steam, which condenses and becomes visible as minute drops of water in the clouds of so-called "steam" which rise from the surface.

From these two experiments we learn that evaporation is aided (1) by heating and (2) by removing the vapour above the surface by a draught. We also see the distinction between *evaporation* and *boiling*, viz. that in the *former* the change from liquid to gas takes place at various temperatures from the *surface* only, while the liquid is apparently still, whereas, in *boiling*, on applying more heat the temperature is not raised above a fixed point (water 100° C.) called the *boiling point* and the change from liquid to gas occurs throughout the liquid, the gas passing through the liquid and escaping. Notice the analogy between *boiling* and *melting* as regards the temperature remaining unchanged although heat is applied ("Latent Heat," §§ 132, 139).

Distillation, Condensation and Liebig's Condenser (see *Exp. Science*, II, Chemistry, § 17).

138. *Exp. To find the Boiling Point of various liquids.

(i) **Ether.** Place about 10 c.c. of ether in a small flask fitted with a cork through which are inserted a thermometer and a delivery tube (Fig. 165). The thermometer bulb must not be in the liquid. Put the flask and ether[1] aside. Warm some water in a beaker to about 60° C. and **put out the flame. Hold the flask in the water and thus boil the ether. Take the temperature when it becomes constant at the B.P. Cool the flask under the tap and put back the ether into the store bottle before lighting the flame again.**

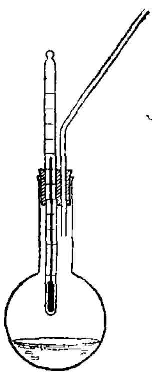


Fig. 165.

(ii) **Alcohol.** (Use methylated spirits.) Repeat the above Exp. with alcohol[1], but keep a small flame under the beaker. The water should be heated until the alcohol begins to bubble freely.

Note. In both these experiments the *hypsometer* (§ 106) may be substituted for the flask shown in Fig. 165.

[1] *Ether vapour* and *alcohol vapour* are highly *inflammable*, therefore the end of the delivery tube should be kept away from a flame.

139. To find the Latent Heat of Vaporization (steam) L.

***Exp.** (1) Weigh an empty ca[illegible] ther with a short *piece of copper tubing* (μ) **87·62** grams.

Fill the calorimeter about [illegible] full of water and

(2) Weigh calorimeter + tu[illegible] **192·87** grams.

(N B. Do [illegible] wet the copper [illegible]

[illegible] water (m) is **104·95 grams.**

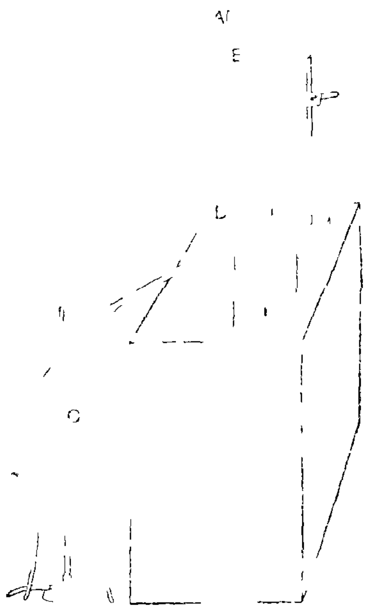


Fig. 166

Clamp a wide glass tube AB (Fig. 166) in a retort stand and fit a rubber cork, with two holes, at B. Through one of these holes a delivery tube for steam passes from the boiler. *Half way* through the other hole a long glass tube EF passes, so that when required, the *copper tube* can be fitted into the *same* hole, the copper and the glass tube thus meeting half way through the

cork. *Remove* the cork at *A* and pass steam through the wider tube to heat it thoroughly; place the calorimeter underneath it. Fix the copper tube in the rubber cork as explained, so that it dips under the water (see Fig. 166) and *note the temperature*, $t°$ ....**18·3° C.**

Close *A* with a cork—the steam now passes at 100° C. into the calorimeter Let the temperature of the water *rise to about* 40° *C*., then remove the cork at *A*, and, with a pair of crucible tongs, draw the copper tube out of the rubber cork, and place it gently in the calorimeter. Stir with the thermometer and note the *highest temperature* (θ) ....**40·06° C.**

Weigh the *calorimeter and contents* immediately to prevent loss of weight by evaporation...**196·71 grams.**

$\therefore$ Wt. of *condensed steam* (M) = (196·71 − 192·57) = **4·14 grams.**

The *rise* in temperature of cold water ($\theta - t$) = 40·06 − 18·3 = **21·76° C.**

" *fall* " " condensed steam ($T - \theta$) = 100 − 40·06 = **59·94° C.**

$$\left.\begin{array}{l}\text{Heat given out by steam in con-}\\ \text{densing + condensed steam in}\\ \text{cooling}\end{array}\right\} = \left\{\begin{array}{l}\text{Heat absorbed by water + calorimeter}\\ \text{and tube in rising in temp.}\end{array}\right.$$

$$M \times L + M(T - \theta) = m(\theta - t) + \mu\sigma(\theta - t),$$

$$4{\cdot}14L + (4{\cdot}14 \times 59{\cdot}94) = (104{\cdot}95 \times 21{\cdot}76) + (87{\cdot}62 \times {\cdot}095 \times 21{\cdot}76),$$

$$\therefore\ L = 536 \text{ calories (approx.)}.$$

Careful experiment has shown that **540** calories are required to convert 1 gram of water at 100° C. to steam at 100° C., hence

the latent heat of steam = **540** calories.

The latent heat of vaporization of alcohol is 202, of ether 90, and of sulphur 362. It is therefore evident that in order *to convert a liquid to gas heat energy must be supplied.* If then we cause some liquid to evaporate either by reducing the pressure above it or by blowing dry air over or through it heat energy will be absorbed either from the surrounding objects or from the liquid itself.

140. Exp. i. To show that when a liquid is made to evaporate, without supplying it with heat, its temperature falls.

Pour a few drops of water on the bench, place a thin metal calorimeter containing a few c.c. of ether on the small pool of water, and cause the ether to evaporate rapidly by blowing air from the bellows through the ether. A temperature of −15° to −20° C. may be obtained and the water will be frozen, causing the beaker to stick to the bench.

Exp. ii. To freeze water by its own evaporation.

Evaporation may be made *rapid* by reducing the pressure. Under the glass receiver of an air pump (Fig. 167) place a dish *A* containing glass wool or asbestos fibre on which concentrated sulphuric acid has been poured; a large surface of acid will thus be exposed. Sulphuric acid absorbs moisture very readily. On a glass tripod over the dish put a clock glass (*B*) containing a few c.c. of previously cooled water. Set the pump working. The water vapour liberated will be removed immediately by the acid and evaporation will be so rapid that the water in *B* freezes.

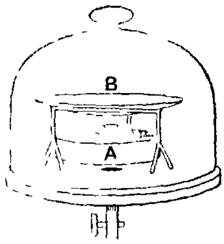


Fig. 167.

Exp. iii. Wollaston's Cryophorus is made as follows.

A glass tube is blown with two bulbs *A* and *B* (Fig. 168), *A* being drawn out into a capillary tube. The bulb *B* is then half filled with water by the same method as was used to fill a thermometer (§ 105). The water in *B* is then boiled and when all air has been expelled the capillary tube at the end

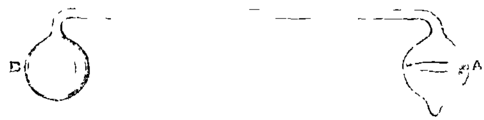


Fig. 168.

of *A* is sealed off. The two bulbs and tube now contain water and water vapour only: the pressure of the air being removed there is almost a vacuum inside the apparatus. The water having been transferred to the bulb *A*, it is cooled down by means of a freezing mixture to, say, 4° C. The bulb *B* is now placed in the freezing mixture so that the water vapour at that end of the tube is condensed; this reduces the pressure still further so that the water in *A* rapidly evaporates until its temperature is so reduced that a crust of *ice* begins to form on its surface.

Freezing machines.

On this principle, freezing machines are made. Liquefied ammonia is generally employed. The hollow walls of a chamber are partly filled with liquid ammonia under pressure. A double pump, vacuum and condensing, reduces the pressure above the ammonia, and causes it to *evaporate*: *consequently heat is removed*

from the chamber. On the other side of the pump a vessel, cooled with running water, receives and condenses the ammonia gas as it is pumped in under pressure. Here is a good instance showing the principle of the **Conservation of Energy**—that **energy cannot be created or destroyed,** it can only be transformed. (1) Work is done *by* the liquid in evaporating, heat must therefore be supplied from itself and the surrounding chamber (2) Work is done *on* the gas in compressing it, heat is therefore developed and is removed by the cooling stream of water surrounding the condensing chamber.

EXAMPLES XXII (LATENT HEAT OF VAPORIZATION).

1. If 1340 calories convert 2·5 gm. of water at 100° C. into steam, what is the latent heat of steam?

2. How much heat will be needed to convert 5 gm. of water at 100° C. into steam? (L. H. = 536.)

3. How many grams of water at 100° C. can be vaporized by 6700 calories?

4. How many calories are required to convert 50 gm. of water at 10° C. into steam?

5. What quantity of water can be raised from freezing point to boiling point by the condensation of 10 gm. of steam?

6. How much steam must be passed into 67 gm. of water at 12° C. to raise the temperature to boiling point?

7. If 936 calories pass into 5 gm. of water at 20° C., how much of the water will be vaporized?

8. What will be the final temperature if 2 gm. of steam are passed into 38 gm. of water at 10° C.?

9. What rise in temperature is caused by passing 20 gm. of steam into a litre of water at 0° C.?

10. Calculate the latent heat of steam if 3 gm. of steam passed into 91 gm. of water at 10° C. raise the temperature to 30° C.

11. 2 gm. of steam are passed into 57·6 gm. of water at 10° C. contained in a calorimeter of water-equivalent 8 gm. If the final temperature is 30° C., what is the latent heat of steam?

12. Calculate the latent heat of steam from the following data: Weight of copper calorimeter = 40 gm.; calorimeter and water = 90 gm.; calorimeter, water and condensed steam = 91·62 gm.; temperature of cold water = 18° C.; temperature after passing in steam = 36° C. (S. H. of copper 0·1.)

13. How many calories will be evolved if 10 gm. of steam at 100° C. are changed into ice at 0° C.?

14. How much heat will be required to change 10 gm. of ice at −10° C. into steam at 100° C.? (S. H. of ice 0·5.)

15. If 2 gm. of steam are passed into 8 gm. of ice at 0° C., what will be the final temperature?

16. Calculate the latent heat of steam from the following: Weight of copper calorimeter = 40 gm.; calorimeter and water = 121 gm.; calorimeter, water and condensed steam = 123·5 gm.; temperature of cold water = 10° C.; final temperature = 28° C. (S. H. of copper 0·1.)

17. How much heat will be evolved if 10 gm. of alcohol vapour condense and the liquid falls to 18° C.? (B. P. of alcohol = 78° C.; S. H. = ·615; L. H. of vaporization = 202.)

18. If 5 gm. of steam are passed into 10 gm. of ice and 55 gm. of water, what is the final temperature?

19. 7 gm. of ice float in water in a calorimeter of thermal capacity 5 calories. If 4·5 gm. of steam raise the temperature to 50° C., how much water was there in the calorimeter?

CHAPTER XXIII.

VAPOUR PRESSURE. WATER VAPOUR IN THE ATMOSPHERE.

141. Vapour Pressures of Liquids.

We have found that a liquid evaporates more readily (*a*) when heat is applied, (*b*) when the vapour above the liquid is removed, and (*c*) when the pressure is reduced. It is therefore natural to suppose that the molecules leaving the liquid exert pressure as gases in general do, and that this **vapour pressure** as it is called varies with the temperature.

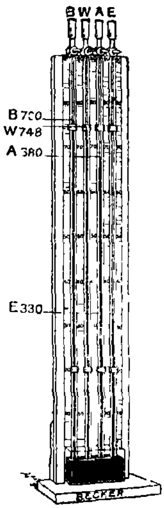


Fig. 169.

Exp. To show that water, alcohol and ether have different vapour pressures.

Four dry barometer tubes are filled with mercury (Fig. 169) and inverted in a bowl of mercury as described in § 57, Exp. vii (*b*). The barometer columns formed should all stand at the same height (say 760 mm.) Each tube is fitted with a funnel and tap at the top. Tube *B* marks the height of the barometer; into the funnel of *W* water is poured, alcohol into *A* and ether into *E*. The taps, which should have previously been turned off with the hole in their plugs filled with mercury, are now carefully turned until a few drops of the liquids are drawn into the tubes and evaporate into the vacuum. If the temperature of the room is 15° C , the column containing

water drops about 12 mm., that containing alcohol about 80 mm. and that containing ether about 430 mm. These numbers measure the vapour pressures respectively.

To show that the **vapour pressure increases when the temperature rises**, very slightly warm the tubes by bringing a flame or a hot body near the liquids in the tubes. An immediate fall of the column of mercury is evident.

142. To determine the vapour pressure of a liquid at various temperatures

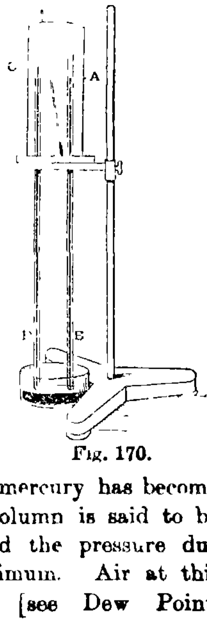


Fig. 170.

About the year 1850 Regnault, a French physicist, accurately determined the vapour pressure of water. One of his methods is shown in Fig. 170. Two barometers *AB* and *CD* are erected and the upper part surrounded by a glass reservoir in which water at various temperatures may be placed. By means of a curved glass pipette, a few drops of water are introduced at the bottom of the tube *AB*. The water rises and evaporates into the vacuum, sufficient water being present to provide a layer of the liquid above the mercury. The depression of the mercury gives the vapour pressure at the particular temperature under observation: when the level of the mercury has become stationary, the space at the top of the column is said to be **saturated with aqueous vapour,** and the pressure due to the vapour present is then at a maximum. Air at this temperature cannot hold more vapour [see Dew Point, § 146].

Ex. From the following results obtained by Regnault, plot a curve on squared paper showing the increase of vapour pressure with the temperature in a space ***saturated with aqueous vapour [maximum vapour pressure].***

Temp. ° C.	Max. Vap. Press. mm. mercury	$t°$ C.	p mm. mercury	$t°$ C.	p mm mercury	$t°$ C.	p mm. mercury
0	4·6	20	17·4	70	233	99	733
5	6·5	25	23·5	80	355	100	760
9	8·0	30	31·5	90	525	101	788
11	9·8	50	91·0	95	634	102	816
15	12·7	60	148·9	98	707	103	845

N.B. The vapour pressures at the higher temperatures given in the above table were obtained by the method of § 144.

143 Exp. i. **To show the increase of aqueous vapour pressure with rising temperature until the boiling point is reached when the vapour pressure = the pressure of the atmosphere.**

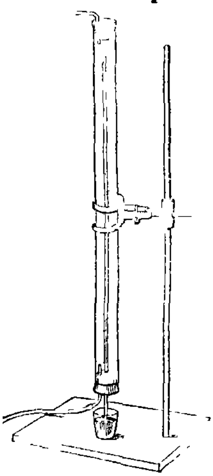


Fig. 171.

A barometer with water in the vacuum space is set up as in the last experiment but the tube is completely surrounded with a jacket, Fig. 171, through which water is passed, the temperature being gradually raised until it reaches about 90° C., when the water is run out and steam at 100° C is passed in. The mercury column gradually falls according to the pressure and temperature given in the table of the last exercise, until the boiling point is reached, when the mercury inside the column is at the same level as that in the trough.

Exp. ii. (*a*) **To show that when a liquid boils its vapour pressure equals the pressure of the atmosphere above it; (*b*) to find the B.P. of alcohol by the method of this experiment.**

(*a*) A J-tube *ABC* closed at *C* contains the liquid (in this case water) and mercury (Fig. 172). The tube is first filled with mercury, closed with the thumb and inverted several times to remove air bubbles. A small quantity of boiled water is poured on to the mercury at *A* until the tube is

full of water and mercury. The tube is then closed with the thumb and inverted until a small quantity of water passes into the short arm, when mercury is poured out of *A* until the level at *B* is below that of *C* as shown in the diagram. The J tube is then heated in a flask by the steam from boiling water when the mercury falls to the same level on both sides of the tube, showing that the vapour pressure in *C* at the B.P. = the pressure of the atmosphere.

(*b*) Introduce alcohol into *C* (instead of water) by the same method; place the tube in warm water and gradually raise the temperature until the mercury is at the same level on both sides. Note this temperature, which is the B.P. of alcohol. Explain why.

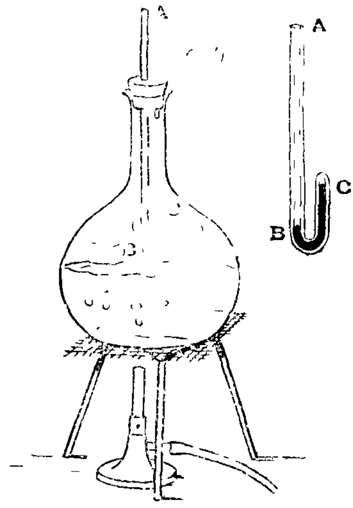


Fig. 172.

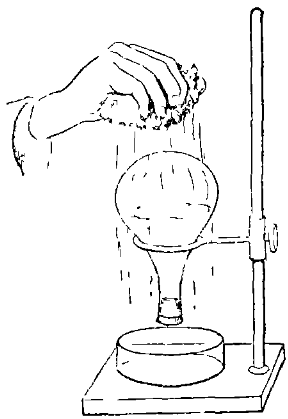


Fig. 173.

***Exp. Franklin's experiment of boiling water at reduced pressure.**

Boil water in a *round bottomed* flask, and while it is still boiling put in a tightly fitting rubber cork, at the *same time* removing the flame. Cool the flask by blowing on it; the water continues to boil. Invert the flask as shown in Fig. 173, and cool it with water. The boiling continues, but finally ceases when the water inside is quite cold. Explain the reasons for this curious behaviour.

144. Exp. Regnault's second method of finding the vapour pressure at various temperatures.

Set up a retort W or a distilling flask (Fig. 174) in connection with an air pump. A thermometer T is inserted into the boiler. By a T-tube, a trap for catching condensed steam and a pressure gauge LR is fitted to the apparatus. Let the height of the barometer be $H=760$ mm. Heat the water W until it nearly boils, then turn the flame low and set the vacuum pump to work. The mercury rises in the gauge to a height $h=$ say 235 mm.

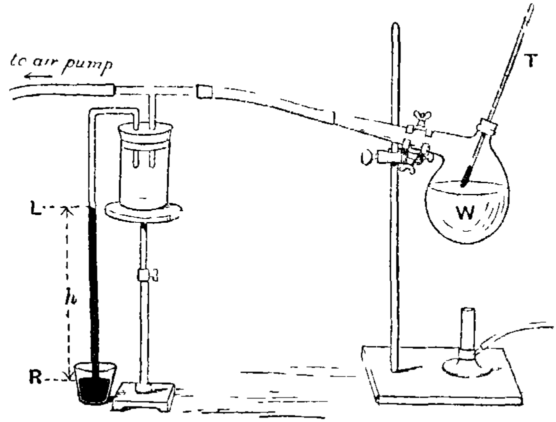


Fig. 174.

when the water begins to *boil*, the temperature being 90° C. The aqueous vapour pressure then equals the pressure on the surface of the liquid, = the pressure inside the tube at L = pressure at R – pressure due to $h=760-235=525$ mm. (cf. table in Ex., § 142). Lower the flame still further. The B.P. falls and the column LR rises. Note h when T registers 80° C. $h=405$ mm. Therefore pressure $=760-405=355$ mm. Since water boils at 80° C. when the pressure on its surface is 355 mm., the vapour pressure of water at 80° C. must equal 355 mm.

Owing to the irregular action of the air pump a more elaborate piece of apparatus constructed on the same principle may be employed (Fig. 175). The flask W and the thermometer T are similar to the retort shown in Fig. 174. A Liebig's condenser C is placed in a sloping position so that the water distilling over runs back into W. A Bunsen's vacuum pump P is

connected (*a*) to the flask by a tap *E*, (*b*) to a large air flask *F*, which prevents the pressure from changing rapidly and (*c*) with a pressure gauge *G* which is filled with a moveable reservoir *R* whereby the readings on the scale may be readily adjusted. A 3 way tap at *D* connects with the atmosphere if necessary or cuts off the pump if pressures above that of the

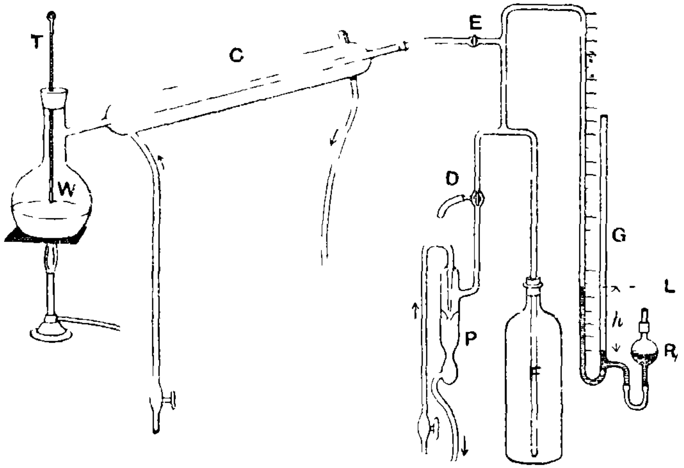


Fig. 175.

atmosphere are required, in which case *R* must be *raised* until the level *L* is brought to the bottom of the scale The difference in level, *h*, must then be *added* to the pressure of the atmosphere (H) In the diagram however the pressure in W and $F = H - h$; the water in W is therefore boiling below 100° C. at reduced pressure.

***Exp. To find the Boiling Point error of a Thermometer.**

The B P. of water is 100° C when the Barometric Pressure = 760 mm. A difference of 26 8 mm in pressure causes a change in the B P. of 1° C. (at or near 100° C)

Read the barometer and *calculate* the *true temperature* (t_t) of the steam from water boiling at the actual atmospheric pressure

Find *practically* (using a hypsometer) the temperature of the steam ($t°$).

Then $t_t° - t° =$ the Boiling Point Error of the Thermometer.

Correction in volume for moist gases.

Towards the beginning of last century John Dalton discovered and formulated the Laws of Partial Pressures in a mixture of gases. He showed that each gas exerts its own pressure in proportion to the volume present, provided that the gases have no chemical action on each other, and that the total pressure is the sum of the partial pressures. Thus the pressure which each gas exerts is the same as if it *alone* were occupying the space.

An example is given in *Exp. Sc.* II, Chemistry, § 9. If 1000 c.c. of a gas were collected over water at say 20° C. and 760 mm. pressure, the space being saturated with aqueous vapour, the pressure due to the gas alone is not 760 mm., but 760 mm. – the maximum aqueous vapour pressure at 20° = 760 – 17·4 = 742·6 mm. (see table, § 142). The corrected and actual volume of *dry* gas at 20° C. and 760 mm. $= \frac{1000 \times 742 \cdot 6}{760} = 977$ c.c.

145. Aqueous vapour in the Atmosphere.

Very little proof is required that there is water vapour in the air. We need only to remind ourselves of the formation of dew, of clouds, of mist, of fog, and of rain—that moisture deposits on the cold windows and walls of hot rooms and on cool objects such as a glass containing cold water.

Hygrometry is the part of physics which deals with the measurement of water vapour in the air.

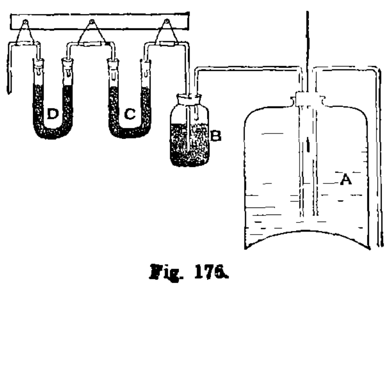


Fig. 176.

Chemical Hygrometer.

Exp. To find the weight of water vapour in a given volume of air. Fig. 176 shows the apparatus used. A is a large vessel, called an *aspirator*, which contains about 10 litres of water, which is slowly siphoned out of the vessel, air being drawn in through the tubes D, C and B to take the place of the volume of water (V litres) removed. The U-tubes D and C are filled with fused calcium chloride which absorbs water vapour and are weighed before and after the experiment, the increase in weight being the weight of water vapour contained in V litres of air. The tube B contains beads moistened with concentrated sulphuric acid which absorbs any water vapour which might diffuse back from A into the weighed U-tubes.

146. Dew Point and Relative Humidity.

In the experiment of § 141 a liquid is evaporated into the vacuum of a barometer tube. We noticed that at a particular temperature there was a definite depression of the mercury column. The space became full of vapour—it was **saturated.** A rise in temperature caused further evaporation to take place and again the space became *saturated with vapour* at a greater vapour pressure. On the other hand, less vapour pressure was required for saturation when the temperature was lowered, and some of the vapour condensed to increase the liquid standing on the top of the mercury So in nature, warm air requires a greater vapour pressure to saturate it with moisture and cool air less. If the air is saturated with aqueous vapour, a fall in temperature causes moisture to be deposited; on the other hand, if the temperature is raised the air is not saturated until more evaporation has taken place and the vapour pressure is increased.

If the air is not saturated at a given temperature, say $t°$, but on cooling it to say $t_D°$ moisture begins to deposit, *i.e.* the air is saturated at $t_D°$, this temperature is called the **Dew Point.**

The **Dew Point** then is the temperature at which the water vapour actually present in the air saturates it. We have therefore only to read from Regnault's tables (§ 142) the maximum aqueous vapour pressure at the dew point ($t_D°$) in order to obtain the pressure of vapour actually present at $t°$.

The Ratio

$$\frac{\text{actual pressure of aqueous vapour at } t^\circ}{\text{maximum pressure of aqueous vapour at } t^\circ}$$

is [illegible] **Relative Humidity**

$$= \frac{\text{maximum vapour pressure at dew point}}{\text{maximum vapour pressure at temperature of atmosphere}}$$

$$= \frac{\text{mass of water vapour actually present}}{\text{mass of water vapour when same volume of air is saturated}}$$

(approx.).

The *wetness* or *dryness* of the air therefore depends on the value of this ratio and not on the actual mass of water vapour present. For instance, since warm air is capable of holding more aqueous vapour than cold air, volume for volume, the addition of a certain amount of moisture sufficient to saturate cold air would not necessarily saturate warm air, although the actual amount of moisture present in the warm air might be greater than that present in the cold air. The "degree of dampness" of the air depends on the quantity of moisture which it can still take up before it is saturated and on the number of degrees it must be cooled before the vapour actually present saturates it.

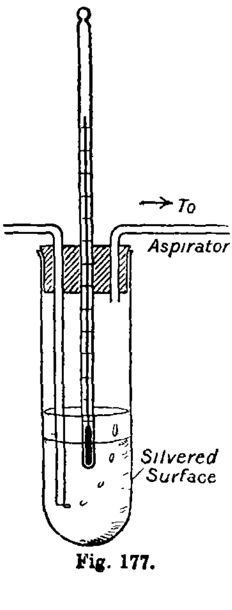


Fig. 177.

147. *Exp. To find the Dew Point.

(1) **Regnault's Hygrometer.** A thin silvered test tube (Fig. 177) is fitted with a cork through which pass a sensitive thermometer and two tubes, one of which passes to the bottom of the test tube. Ether is placed in the test tube and air is bubbled through it by the longer of the two tubes. The shorter tube is attached to an *aspirator*. The temperature

falls as the ether evaporates, and is noted when a deposit of dew is formed on the silvered surface—this temperature is the dew point.

(2) Metal Cup Hygrometer. The method is similar to (1), **but water is placed in a highly polished metal cup and cooled by the addition of small pieces of ice. The water is stirred and the temperature noted when a deposit of dew appears. Care should be taken not to breathe on the cup.**

Temp. of air	15° C.	Max. Vap. Press. at 15° C. = 12·7 mm.
Dew point	5° C.	,, ,, 5° C. = 6·5 mm.

$$\therefore \text{ Rel. Humidity} = \frac{\text{Actual Vap. Press at } 15^\circ \text{ C.}}{\text{Max. Vap. Press. at } 15^\circ \text{ C.}}$$

$$= \frac{\text{Max. Vap. Press. at } 5^\circ \text{ C.}}{\text{Max. Vap. Press. at } 15^\circ \text{ C}} = \frac{6 \cdot 5}{12 \cdot 7} = 0 \cdot 52.$$

Dine's Hygrometer (Fig. 178 *a*). Water cooled with ice is stored in a box (B) and passes in a stream regulated by a tap (S) through a space containing a thermometer (TT). The bulb of the thermometer is placed under a piece of blackened glass (G) which forms part of the upper wall of the space. When a deposit of dew first appears on the plate (G), the temperature (t_1°) is noted and the stream of water is stopped. The temperature is again taken when the dew disappears (t_2°). Then $\frac{t_1^\circ + t_2^\circ}{2}$ is the **dew point.**

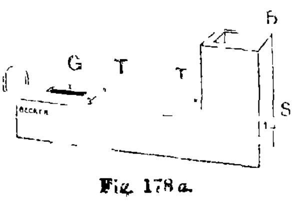


Fig. 178 *a*.

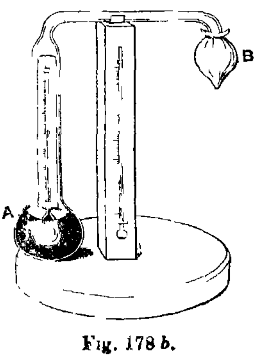


Fig. 178 *b*.

Daniell's Hygrometer is shown in Fig. 178 *b*. It consists of a Wollaston Cryophorus containing ether instead of water. The bulb *A* is silvered or gilt within and contains a thermometer. The bulb *B* is covered with muslin over which ether is dropped. This ether evaporates and there is a consequent cooling of the bulb *B*. Evaporation takes place from *A* into *B* (§ 140) and *A* becomes colder until the dew point is reached, when the polished surface of *A* becomes dulled.

Mason's Hygrometer, shown in Fig. 179, is also called the **wet- and dry-bulb thermometer.** *A*'s bulb is kept moist by lamp wick dipping into a cup of water. Evaporation into the atmosphere cools *A*. If the atmosphere is very dry the temperature of *A* will be several degrees below that registered by *B*, which is the actual temperature of the air. A set of hygrometric tables have been drawn up for use with the hygrometer, in which opposite to each degree of temperature registered by the dry bulb the pressure of aqueous vapour is recorded according to the number of degrees difference between the readings of the wet- and dry-bulb thermometers.

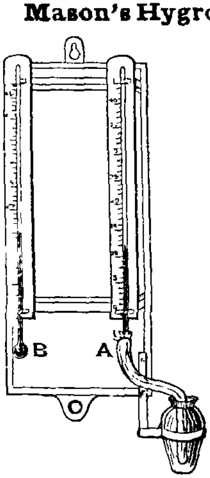


Fig. 179.

Example. Reading of *dry* bulb 15° C. } Difference 4° C.
,, *wet* ,, 11° C. }

From the tables the pressure of aqueous vapour at 15° C. for a difference of 4° C. between wet- and dry-bulb thermometers is 7·4 mm.

$$\therefore \text{ Rel. Humidity} = \frac{7 \cdot 4}{12 \cdot 7} = 0 \cdot 58.$$

148. Clouds. The prevailing winds in the British Isles are from the south-west. The moisture-laden air warmed through its contact with the drift current from the Gulf Stream is cooled owing to several causes. (1) The air is moving from warmer to

colder latitudes; (2) it strikes the land which by night, at least, is colder than the sea; (3) it is forced up into higher and colder altitudes, and (4) as it rises it expands and consequently cools.

Moist air in contact with the earth on being warmed by the sun rises, warm air being less dense than cold air. The water vapour contained in the air tends to condense in the form of cloud for reasons (3) and (4) given above. The minute drops run together and fall as rain, which may reach the ground or it may pass through warmer or drier layers of air and completely evaporate in its descent.

Hail is frozen rain, but its exact mode of formation is uncertain. It is supposed to come from great heights. **Snow** is the result of a deposit of moisture taking place at a temperature below 0° C.

Mist is the formation of cloud in a valley, or on the ground. It usually occurs in still air over lakes, rivers or marshes which have been warmed during the day. The air above them is charged with water vapour which condenses as the sun goes down. **Fog** is mist where each little globule of water has formed around a particle of dust or soot as a nucleus.

Dew is a deposit of moisture on objects, cooled by radiation, during clear still nights. The moisture comes in part from the atmosphere and in part from vegetation and the damp ground beneath. When rapid radiation has cooled the air below 0° C., condensation takes place directly from vapour into ice, without the intermediate watery stage, and **hoar frost** or **rime** results.

Examples XXIII a (Vapour Pressure).

1. How would you cause water to boil (*a*) at 60° and (*b*) at 100° C.? What would be the pressure when the B.P. is 60° C.? At what temperature would water boil on the top of Mount Blanc if the barometer stood at 430 mm.?

2. What difference in volume would be caused in 500 c.c. of dry air (free to expand or contract) at 700 mm. pressure and 20° C. if a few c.c.s of water were introduced without altering the pressure and the temperature?

3. In a chemical experiment 500 c.c. of gas was collected over water at 30° C. and 790 mm. What volume will the dry gas occupy at 760 mm. and 0° C.?

4. It is desired to infer from a volume measurement the mass of a quantity of air confined in a graduated tube over water. What observations must be made? O. J.

5. How is the saturation pressure of aqueous vapour at ordinary temperatures measured? A little water lies on the top of the mercury column in a barometer. What error would be made if this barometer were used to measure the pressure of the atmosphere? Would the error be the same on a cold day as on a warm day? C. J.

6. Describe experiments which show that (*a*) the vapour pressure of ether is greater than that of water, (*b*) that the vapour pressure of water at boiling point is equal to atmospheric pressure. O. & C. J.

7. Calculate the boiling point error of a thermometer which registers 99·8° C. in steam when the barometer reading is 747·9 mm.

EXAMPLES XXIII B (HYGROMETRY).

1. Account for the fact that, when a glass of cold water is brought into a hot room, moisture condenses on the outside of the glass. (O. J.) Why does not this occur usually on a cold winter day?

2. How would you determine the amount of water vapour actually present in a given quantity of air?

Sketch the apparatus required.

3. What is meant by the "hygrometric state" of the air?

Describe a method of determining it, showing exactly how it may be calculated from the observations made? O. J.

4. What is meant by "Dew point"? Describe Daniell's hygrometer and explain clearly the phenomena that occur when it is used for ascertaining the dew point. O. J.

5. Explain why the reading of a thermometer whose bulb is covered with a wet cloth is usually different from that of another placed near it whose bulb is bare.

State, giving reasons, under what circumstances you would expect the difference to be increased or diminished. O. J.

6. Why would you expect (*a*) fogs round icebergs, (*b*) a morning fog to clear before mid-day?

7. Sketch and describe Dine's Hygrometer.

In what way is it an improvement on Daniell's Hygrometer?

8. Explain how (*a*) the relative humidity and (*b*) the dew point may be determined from the readings of the wet- and dry-bulb hygrometer. Of what practical value in every-day life is the determination of the relative humidity?

9. What are the most favourable conditions for the heavy deposit of dew.

Describe the simple forms of hygroscopes O. J.

10. The S.W. wind prevails in England and generally brings rain. Account for this. O. J.

CHAPTER XXIV.

CONDUCTION AND CONVECTION.

149. Heat may leave a body in three ways: (*a*) by conduction, (*b*) by convection and (*c*) by radiation.

Conduction. The handle of the tea-pot, the kettle or the saucepan is hot because **heat is conducted** along the metal from the boiling water inside the vessel. *Metals* are *good conductors* of heat. A wooden handle is fitted to the saucepan, or a woollen kettle holder is used so that we can take hold of the hot metal, because *wood* and *wool* are *bad conductors* of heat.

Convection. If we heat an iron ball red hot and suspend it by a fine wire, very little heat leaves the iron by conduction because the channel through which heat is conducted, viz. the wire, is thin. A lighted taper held above the heated iron flickers; it is burning in a current of air heated *by contact* with the hot ball, each particle of air carrying away heat energy; the air expands on being heated, is rendered less dense and consequently it rises. Heat is being removed by a **convection current** in the air rising from the ball.

Radiation. But we also feel the heat from the iron if we place our hands *around* or *below* it. If we hang up the red hot ball under the receiver of an air pump and exhaust the air, yet the ball gives out heat. Heat energy can be carried across a vacuum and through the glass by **radiation.** Radiant energy from the sun, in the forms of heat and light, traverses millions of

miles being conveyed by the *ether* of space. All the heavenly bodies and the earth itself are cooling through loss of heat into space by radiation. *Radiant heat does not warm the medium* through which it passes.

(A) CONDUCTION.

150. Relative Conductivity.

To show the Relative Conductivity of several substances.

Fig. 180 shows a method of comparing the conductivities of Silver, Copper, Brass, Aluminium, Zinc, Wrought Iron, Steel, Glass and Wood. Rods (*C*) of the *same length and thickness* are fitted through corks into a trough *AB* into which boiling water may be placed. Before adding the hot

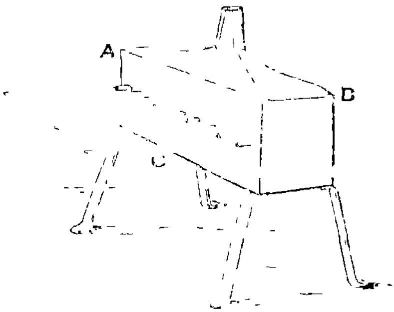


Fig. 180.

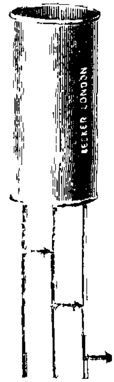


Fig. 181.

water the apparatus is prepared by dipping the rods in a bath of melted paraffin wax so that they are coated equally. The rods are protected from radiation from the hot tank by a perforated cardboard shield, not shown in the figure, through which the rods protrude. The metals are given above in the order of their conductivities, which are proportional to the square of the length of wax melted along the rod. The wax on the glass and wooden rods is scarcely affected.

The above experiment may be shown more graphically by using a device suggested by Mr Edser. The rods are placed vertically (Fig. 181) and each

is fitted with a ring to which a pointer is attached. The ring is coated with paraffin by which it is held in position, touching the can into which hot water is poured. As the wax melts the ring and index fall down the rods by their own weight until they are arrested by the wax congealing on the cooler part of the rods. The rods shown in Fig. 181 are (reading from left to right) steel, brass and copper.

151. Comparison of Conductivities of Brass, Copper and Iron by sensitized paper[1].

***Exp.** Three wires about 14 inches long and $\frac{1}{16}$ inch diameter of the different metals are bent at right angles (*A*) near their ends and placed as shown in Fig. 182 on a piece of sensitized paper supported on a board. The

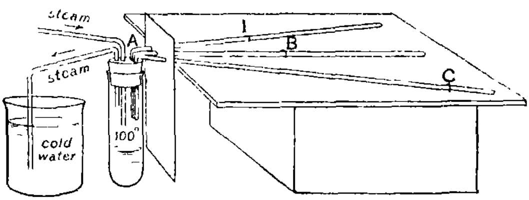


Fig. 182.

wires dip into a boiling-tube through which passes a current of steam from a boiler. A piece of cardboard perforated with three holes protects the paper from the source of heat. After heating at 100° C. for about 20 minutes a steady temperature is obtained. Lift each wire and mark with a pencil (*C*, *B*, *I*) the end of the green mark. *Measure the distance of C, B and I from the right angle bend A* = (say) Copper 10, Brass 5·5, Iron 4·5; then their relative conductivities are approximately as the *squares* of these numbers, *i.e.* 100 : 30 : 20.

[N.B. If preferred the beaker may be replaced by a crucible containing *mercury* heated by means of a burner and a copper rod (see Fig. 184).]

152. Exp. The rate at which a piece of metal

becomes heated when placed in contact with a hot body depends partly on

[1] *Sensitized Paper*—prepared by soaking sheets of thick filter paper or cartridge paper in a solution of *Cobalt Chloride* or *Bromide*. The sheets should be slowly but thoroughly dried. *Heat* turns them a light *apple-green* colour, but on cooling in a *moist* atmosphere the original condition is restored.

its *conductivity* and partly on its *specific heat.* Fig. 183 shows a rod of lead and a rod of iron of the same size protruding from a trough. Marbles are attached to the rods at equal intervals by paraffin wax. When hot water is poured into the trough the marbles fall at first more rapidly from the lead

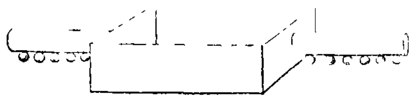


Fig. 183.

rod, but ultimately a greater number are melted off the iron rod. *Lead,* having a *lower specific heat* (·031) than iron (·113), absorbs less heat in being raised to a certain temperature, hence the lead is more rapidly heated, but as its conductivity is less than that of iron, ultimately the paraffin is melted to a greater distance along the iron bar.

The action of Wire Gauze in conducting heat from a flame and its application to **the Davy Safety Lamp.** See *Exp. Science* II, Chemistry, § 115.

153. To prove that water is a bad conductor of heat.

***Exp.** Put a piece of ice into a large test tube and drop in on the top a piece of crumpled wire gauze which just fits the tube in order to keep the ice at the bottom. Nearly fill the tube with water, and by playing the Bunsen flame on the upper part of the tube boil the water near the surface while the ice remains unmelted at the bottom of the tube.

To show that mercury is a good conductor of heat.

Wrap a piece of *sensitized paper* round a test tube containing *mercury* into which a thick copper rod is dipping (Fig. 184). Heat from a Bunsen burner is conducted by the rod to the upper surface of the mercury. A green colouration spreads downwards on the paper, showing that the liquid metal is a good conductor of heat.

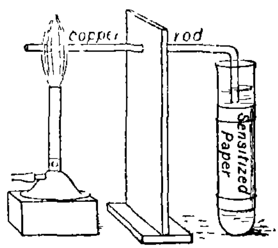


Fig. 184.

154. Gases are bad conductors of heat.

The warmth of our clothes depends not so much on actual weight of material as on the amount of air which they keep enclosed in their texture. Thus a loosely woven cloth is often oppressive to wear when there is no wind because of the envelope of air with which it surrounds our bodies.

Birds on cold days fluff out their feathers to make a thicker coat, not of feathers, but of air.

In a cold country double windows and doors are fitted in the houses because the enclosed air is a bad conductor of heat.

If we wish to keep ice from melting on a hot day, we wrap it in loose flannel.

155. Absolute Conductivity.

The amount of heat which passes in 1 second through a cubic centimetre of the material when the difference in temperature between two opposite faces of the cube is 1° C. is a measure of the absolute conductivity of the material.

Imagine for instance that we have a plate of copper 1 *cm thick* and that we mark out square centimetres, one on each side of the plate, so that if we joined the corners of these squares we should form a *cubic centimetre* (Fig. 185) and that we maintain the temperature of one side of the plate at 1° C. above that of the other side, then the number of calories carried through the cubic centimetre in *each second* is called the **Coefficient of thermal conductivity** (k).

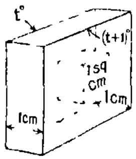


Fig. 185.

If Q = quantity of heat transmitted through a plate of A sq. cm. *area*, and of *thickness* d cm., in t *seconds*, when the difference in temperature is $\theta°$ between the two sides, then, if we make A, d, t and θ all equal to unity, we obtain the value of k, then

$$k = \frac{Q \times d}{A \times \theta \times t}.$$

Coefficients of Thermal Conductivity (in Calories).

Silver	1·33	Water	·002
Copper	0·98	Glass	·0005
Brass	0·32	Wool	·00012
Iron	0·20	Air	·00005

(B) CONVECTION.

156. When a liquid or a gas is heated it is almost impossible that the temperature of the whole mass should rise equally throughout the fluid. Consequently differences of density occur according to differences in temperature. The colder and denser parts tend to sink, the hotter and less dense tend to rise. Thus a circulation takes place and currents are set up by which the heat is distributed more evenly. It should be noted that the *heat is carried by the substance of the fluid actually moving from one place to another, the movement being due to difference of density.*

To show convection in liquids.

***Exp.** i Put small pieces of filter paper or pour a little aniline dye dissolved in spirit, carefully, on to the surface of cold water contained in a large beaker. Warm the water by a flame concentrated as much as possible under *the centre* of the beaker. Describe exactly the direction of the convection currents set up in the water.

***Exp.** ii. Float a large lump of ice *at one side* of a large beaker of tepid water; when the water is apparently still drop in aniline dye near the ice. Describe and explain what you see take place.

Hot water apparatus. (Model shown in Fig. 186.)

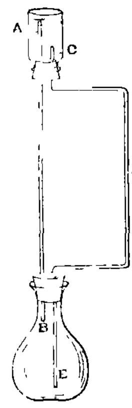


Fig. 186.

Exp. *AC* is a corked funnel open at the top connected by tubes *AB* and *CDE* to a flask. Arrange that the upper end *A* of *AB* is above *C*, and that the lower end *B* is above *E*. Pour water into the funnel *AC* and completely fill the

apparatus. Put some magenta dye into AC and warm the lower flask with the Bunsen flame. Convection currents are set up which move in the direction $BACDE$. This is a model of the apparatus used for heating buildings by hot water pipes and radiators.

Ocean Currents. The surface currents of the oceans are mainly due to the prevailing winds, but there is a much vaster though slower movement of the waters at great depths towards the equator. This general abysmal current is known as "the ocean creep." As the equatorial regions are warm and the polar regions cold, there tends to be a flow of warm surface water northward and southward from the equator to take the place of the colder water which sinks to the bottom of the oceans in higher latitudes.

The Gulf Stream is not a deep seated current; it is for the most part a surface drift-current caused by the Trade winds carrying the warm waters of the Tropics towards the southern part of the Gulf of Mexico. The configuration of the land causes this warm stream to issue south of Florida and the prevailing south-westerly winds carry it across the Atlantic towards Britain and Norway. Consequently the climate of England is much milder than that of Labrador at the corresponding latitude on the other side of the Atlantic, where a southerly current carries drift ice from Baffin's Bay.

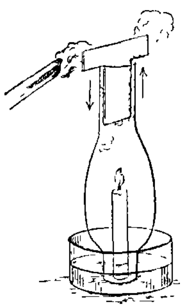


Fig. 187.

157. To show convection in gases.

***Exp. i.** Place a large lamp chimney over a candle burning in a dish containing a little water. Put into the chimney a sheet of tin cut in the shape of the letter T (Fig. 187). Place a smouldering piece of brown paper near the top of the chimney and notice that there is an in- and down-current and an out- and up-current on opposite sides of the tin.

Exp. ii. Model to show ventilation in mines. Fig. 188 shows a box connecting two chimneys, at the bottom of one of which a candle is burning. The smoke from smouldering paper is carried down the chimney, which has no candle at its base. In some

mines, two shafts are sunk and a fire is kept burning at the base of one of them. A powerful current of air ascends this "up-shaft": it is found that the deeper the shaft, the stronger the current becomes. The "down-shaft" is used for raising the coal to the surface.

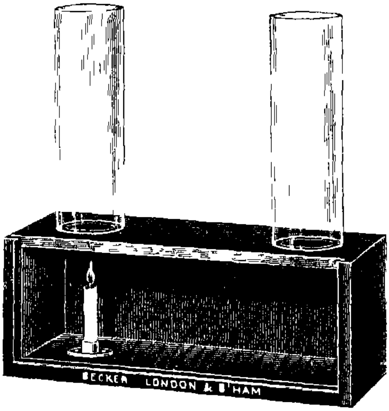


Fig. 188.

Ventilation of Rooms is often effected by the draught up the chimney when a fire is burning. Large halls are sometimes ventilated by removing the hot, impure air which rises towards the roof, by a central shaft, communicating with the outside, in which a row of Bunsen burners is kept lighted.

Winds. Over a heated area of the earth there is always a rising current of air. Towards this heated area there is an inrush of air near the surface of the earth. The **Trade Winds** for instance are caused by rising currents of air over the heated equatorial regions causing an inrush near the ocean from southern and northern regions. If the earth were a vast plane this inrush of air on either side of the equator would cause winds from due north and due south. But the earth is a sphere rotating on its axis and the parts nearer the equator, because they describe greater circles of latitude, are moving faster than those to the north and south. The earth rotates from *west to east*, consequently a spectator facing *east* partially meets this inrushing

wind, much in the same way as a cyclist who finds when he is moving that there is an *opposing* wind, although when he was standing still its direction was *across* his path. Hence in the northern hemisphere the trade winds are from the N.E. but in the southern hemisphere from the S.E.

Monsoons. As stated above there is always an inrush of air in the lower regions of the atmosphere towards a heated part of the earth's surface. In summer Central Asia is a hot region; on the other hand the Indian Ocean, owing to its high specific heat, is comparatively cool (§ 129). Steady winds consequently blow from the Ocean towards the continent. In winter however the reverse takes place as the land loses its heat rapidly through radiation and the sea is relatively at a higher temperature. The fact that in our northern winter Australia becomes a heated area aids in reversing the southern monsoons.

Land and Sea Breezes. Similar changes on a small scale take place in summer round the coast provided there are no great disturbances, such as cyclones, in the atmosphere. By day a breeze comes in from the cooler sea to rise over the heated land, by night a land breeze moves out over the relatively warmer ocean.

EXAMPLES XXIV A (CONDUCTION).

1. What do you mean by "Conduction of Heat"? Give examples to illustrate your answer.

2. Describe an experiment to discover whether iron or lead conducts heat here. O. J.

3. Why is it that to keep a block of ice in warm weather we wrap it up in flannel and to keep our bodies warm we choose flannel clothes?

4. Describe and explain the action of a piece of wire gauze on a flame. How is this action utilised in the safety lamp used by miners? O. J.

5. Three rods of equal size made of copper, iron, and lead respectively were coated with wax and fixed horizontally, each with one end in a trough of boiling water. After a few minutes the wax was seen to have melted

farthest along the lead and least along the iron rod. Later no more wax melted on any of them and it was noted that the copper had least unmelted wax and the lead most. Which metal is the best and which the worst conductor? Account for the difference in the two sets of observations. Why did not the whole of the wax melt on each rod? O. J.

6. Describe an experiment to show that water is not a good conductor of heat, explaining your reasons fully. O. J.

7. In what way could you show that mercury is a good conductor of heat and air a bad one? O. J.

EXAMPLES XXIV B (CONVECTION).

1. Explain what is meant by the convection of heat. A lamp is held under a beaker of water in which some sawdust is placed. Describe the movements of the sawdust. O. J.

2. Account for the fact that water can be boiled in a paper box without the paper being charred.

3. Some water is contained in a glass cylinder and is (*a*) heated for some time at its upper surface, (*b*) cooled for some time at its upper surface. State and explain what happens in each case. O. J.

4. Apply the principle of convection to account for the fact that at many places in the tropics the wind blows from the sea to the land during the day and in the opposite direction during the night. O. J.

5. In order to ventilate a room, should the window be opened at the top or at the bottom? Give reasons for your answer.

6. Describe some method for ventilating mines.

7. Draw a diagram of a hot water system for heating a building, and explain its action.

CHAPTER XXV[1].

RADIATION AND TRANSFORMATION OF ENERGY.

158. The ether of space and the analogy between heat and light.

In § 149 it was pointed out that radiant energy not only travels across a vacuum but also through millions of miles of space from all the heavenly bodies. We know very little about the medium which conveys *radiation*: we call it the "*ether of space.*" It transmits *light* as well as *heat*: it is not affected after transmission has taken place. *Light energy* and *heat energy* are found to be of the same order. Both travel in *straight lines* and with the same velocity (186,000 miles per second). In a solar eclipse the light and heat from the sun are cut off at the same instant and there is a clear and sharply cut shadow for both forms of radiant energy. It has been proved that both are a "mode of motion," transmitted as waves differing only in their wave lengths, *i.e.* in the distance between crest and crest of consecutive waves. Specialized parts of our bodies are affected by vibrations of certain wave lengths. Further analogy between light and heat as regards *reflection* and *refraction* is shown in § 161.

Exp. If a current of electricity is passed through a spiral of thin platinum wire and the strength of the current is gradually increased, the

[1] May be omitted for the first reading.

wire becomes hotter and hotter until it glows with a dull *red* colour, then it becomes *white* hot and finally it melts. Experiment has proved that the vibrations set up in the ether round the wire differ in wave length; as the temperature rises the vibrations increase in rapidity and their wave lengths become shorter, those of approximately 700 to 400 millionths of a mm. in length affect the specialized part of our bodies called the *retina* of the eye and produce what is called *light*, while these as well as the vibrations of longer and of shorter wave-length produce the *sensation of heat*.

159. To show that the radiating qualities of surfaces vary.

***Exp.** Two tin canisters (*e.g.* mustard tins) of *equal size*, the one (*A*) brightly polished and the other (*B*) thickly coated with *lamp-black*[1], have each a thermometer fitted through a wooden lid (Fig. 189). Pour (1) hot water at about 80° C. into *A* and (2) an equal volume at slightly higher temperature into *B*. Note the *time* when both thermometers record the *same* temperature and take *thermometer* readings every half-minute during 10 minutes. Plot temperature/time curves on squared paper to show the relative *rates of cooling* and thus compare the *radiating* qualities of polished tin and lamp-black.

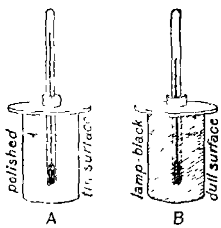


Fig. 189.

(N.B. For use in a subsequent experiment [§ 165] take the temperature of the surrounding air.)

160. To show that good radiators are good absorbers of heat.

***Exp.** In the last experiment we discovered that a dull black surface radiates heat more readily than a polished one. If the same canisters holding equal quantities of cold water at the same temperature are placed for the same length of time at equal distances from a source of heat (say a stove or hot fire) it will be found that the water contained in the canister covered with *lamp-black* absorbs considerably more heat than the water in the *polished* tin. Read the thermometers every 5 minutes (during say 20 minutes) and plot the results on squared paper.

[1] Hold the cold tin in the smoke of burning *camphor* or in a candle flame.

161. Instruments for detecting and estimating Radiant Heat.

(1) **The Ether-Vapour Thermometer** resembles an *air thermometer* (§ 116, Fig. 157) except that the outside of the flask is covered with *lamp-black* (§ 160) and contains a small quantity of *ether* coloured with an aniline dye.

(2) The **Thermopile** consists of several pairs of rods of the metals *Antimony* (*A*) and *Bismuth* (*B*) arranged alternately [1, 2, 3, 4, 5, Fig. 190]. If the ends *A* and *B* of the series of rods are joined by a wire and the junctions 1, 3, 5, &c. are heated, a current of electricity passes through the circuit. If the junctions are cooled the current is reversed. A sensitive *galvanometer* *G* detects the passage of the current and by its deflection measures the relative amount of radiant heat falling on the junctions.

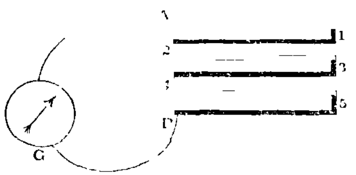


Fig. 190.

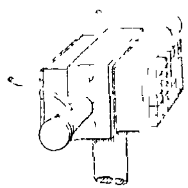


Fig. 191.

A *thermopile* having 36 pairs of rods packed side by side is shown in Fig. 191. The junctions at the ends of the rods are on the right and left in the fig.

162. To show that heat is reflected from a polished surface.

Exp. Draw two dotted lines at an angle of about 90° on the bench and above and parallel with these lines clamp two wide tin tubes (Fig. 192). At

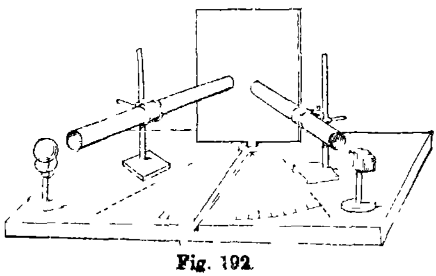


Fig. 192.

the intersection of the two lines place in a vertical position a sheet of *tinned iron* with a pointer attached by which the tinned sheet may be rotated about a vertical axis. Place a *ball of hot iron* and a *thermopile*[1] (or ether-vapour thermometer) at the ends of the tubes as shown in the figure. Move the pointer horizontally so as to rotate the sheet until the *maximum heat-reflection effect* is shown by the thermopile (or thermometer). If the pointer is fitted at right angles to the plane of the sheet, the pointer will be found to bisect the angle between the dotted lines (see figure). Replace the ball with a lighted candle; on looking down the tube with the eye at the position of the thermopile an image of the candle will be seen reflected in the polished tin surface.

Refraction of heat as well as of light is shown when the rays of the sun focused by a lens, used as a burning glass, are concentrated and set fire to a piece of paper or touch wood.

163. Revision Experiments.

(i) **Leslie's Cube** (Fig. 193). The four vertical sides of a cubical tin vessel are prepared as follows: (1) covered with *lamp black*, (2) polished *grey*, (3) painted *white* and (4) highly *polished*. Place a thermopile about 18 inches from the cube. Fill the "cube" with hot water and compare the *radiating qualities* of the four surfaces.

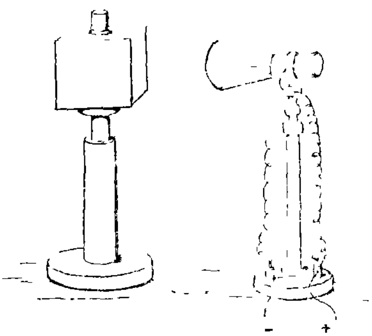


Fig. 193.

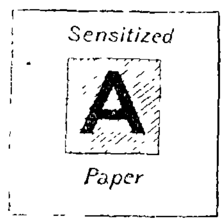


Fig. 194.

(ii) On a piece of *sensitized paper* (§ 151) paint in *dull black* colour a letter A (= *Absorption*) and round this paint, with aluminium paint, a reflecting square as background (Fig. 194). When quite dry bring the painted surface near a source of heat; note and explain the colour changes observed on the back of the sheet.

[1] The galvanometer and wires are omitted from the figure.

(iii) Set up two spherical mirrors opposite each other as shown in Fig. 195. Place a hot metal ball at the *focus* of one mirror and show by the *thermopile* that heat is reflected to the focus of the other. Darken the room

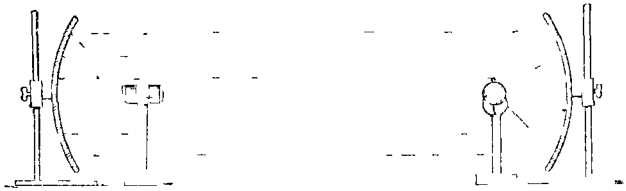


Fig. 195.

and show that if the hot ball is replaced by a candle flame an *image* of the flame may be obtained on a small disc of paper fitted on the end of the thermopile.

164. Diathermancy. (*Transmission of Heat.*)

We know that a sheet of glass is *transparent* to light, and, on the other hand, that a sheet of iron is *opaque.* Experience tells us that most solids and liquids are "opaque" to radiant heat. A sheet of *iron*, for instance, placed before the fire, completely screens us from the heat. A sheet of *glass* interposed partially screens from the warmth of the fire, but a sheet of *rock-salt* scarcely screens at all.

The word corresponding to *transparency* for light is *diathermancy* for heat. *Rock-salt* is almost perfectly *diathermanous*, *glass* is partially so, but *iron* does not transmit heat rays. A solution of *iodine* in *carbon bi-sulphide* transmits heat energy (is diathermanous) but is opaque to light. *Water* transmits light but not heat rays.

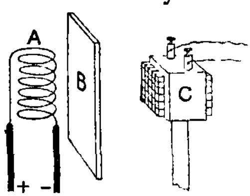


Fig. 196.

To compare the diathermancy (transmissive power) of (*a*) *Rock-Salt* and (*b*) *Glass*.

Exp. The source of heat is a spiral of platinum wire (*A*, Fig. 196) through which a controlled current of electricity is passed. Increasing the current causes the wire to become (1) hot but non-luminous, (2) red-hot,

(3) white-hot. A thermopile (C) measures the relative amounts of heat transmitted. Thin sheets of the substances are interposed at B.

Approximate results are as follows:

Diathermancy of *air* = (say) 10.

Substance	(1) Wire (A) hot but *non-luminous*	(2) Wire (A) *red-hot*	(3) Wire (A) *white-hot*
Rock-Salt	9	9	9
Glass	0	1	8

It is evident then that glass screens the heat derived from a moderately hot body such as red-hot fire but transmits the heat of a hotter body such as the sun to a much greater extent. For this reason a hot-house with a glass roof retains the heat transmitted from the sun.

165. Newton's Law of Cooling.

Write down in columns the results of § 159 to show the rate of cooling of one of the cans of water, as follows:

Temperature of outside air = ° C.

Time—half minute intervals	Temperature of cooling vessel ° C.	Fall in temperature per half-minute	Average difference in temperature between cooling vessel and surrounding air
(1)	(2)	(3)	(4)

From the values obtained in columns (3) and (4) show that *when a body cools it loses in a given interval of time quantities of heat which are proportional to the difference in temperature between the cooling body and the surrounding air*, *i.e.* rate of cooling is proportional to difference in temperature.

166. The Mechanical Equivalent of Heat.

Reference has already been made to the transformation of energy and to the fact that energy cannot be created or destroyed. In the years 1840—1843 James Prescott Joule of Manchester proved by experiment that there is always a definite relationship between the quantities of energy transformed. For instance, if a definite quantity of electric energy is passed through a particular coil of wire a definite quantity of heat is developed in the wire. Or again, if work is expended in stirring water in a calorimeter, the ratio between the work done and heat generated is always constant.

Joule's equivalent. The number of units of mechanical work equivalent to one unit of heat is called *Joule's equivalent* (J) or the *mechanical equivalent of heat.* In Lat. 45° and at sea level, 777 foot-pounds of work are equivalent to the amount of heat required to raise 1 lb. of water 1° F., *or* 426 gramme-metres of work are equivalent to 1 calorie.

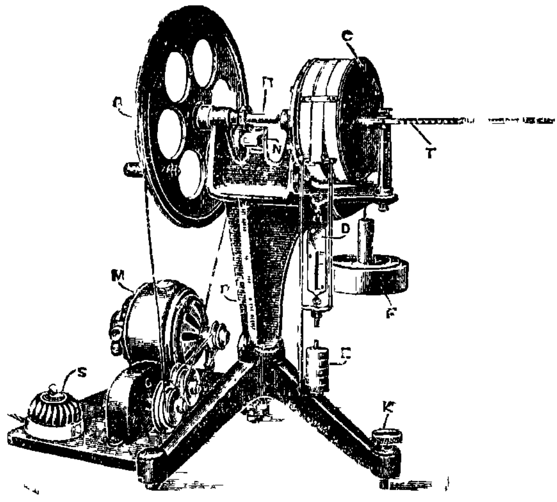


Fig. 197.

To determine Joule's equivalent, "*J*." (*The mechanical equivalent of heat.*)

Professor Callendar's apparatus for determining "J" is shown in Fig. 197. It consists of a drum-shaped calorimeter (C) which is rapidly rotated. A band-brake held tight by weights E and F opposes rotation, and work done against friction raises the temperature of the water and calorimeter. The *work* done is found by multiplying the *force* (F gram) of friction, which is obtained from the difference in tension of the band on each side of the drum, by the total *distance* (s cm.) through which a point on the circumference moves during the experiment. The number of revolutions (n) is read by the mechanical counter (N).

Then $\quad s = 2\pi r \times n$ cm. [where r cm. = radius of drum]

and $\quad$ work done $= Fs = 2F\pi rn$ gram-cm.

An electric motor M rotates the drum at the rate of about 100 revolutions per minute.

The water equivalent (μ) of the calorimeter should be previously determined. About 350 grams (M) of water at about 5° C. *below* the temperature of the laboratory is placed in the calorimeter and motion is continued until the thermometer T registers about 5° C. *above* the temperature of the laboratory. If

$$t^\circ \text{ C.} = \text{rise in temperature,}$$

then $\quad$ Heat developed $= (M + \mu)\, t$ calories,

$$\therefore \text{ Work equivalent to 1 calorie} = J = \frac{2F\pi rn}{(M+\mu)\,t} \text{ gram-cm.}$$

Examples XXV a (Radiation).

1. What do you mean by "Radiation" of heat?

Give examples illustrating your answer.

2. Describe experiments proving that the laws of reflection and refraction of light apply also to heat.

3. How can the emissive (radiating) powers of different substances be compared?

4. A room is heated by a radiator through which hot water circulates. Name all the processes by which heat is transmitted from the fire below the boiler to the radiator and from the radiator to the objects in the room. O. J.

5. How can it be shewn that good absorbers of heat are also good radiators?

Why is glass used in the construction of greenhouses in spite of the fact that a glass screen will protect us from the heat of a fire? O. J.

6. Four similar thermometers are placed as follows: No. 1, on grass: No. 2, above ground, screened but with free access for air; No. 3, unscreened; No. 4, on a wall facing S.W. How would you expect their readings taken (*a*) shortly after sunset, (*b*) at noon, the sky being cloudless, to differ? Account for the difference of the readings. O. J.

7. What do you mean by "Diathermancy"?

Describe an experiment by which the diathermancy of various substances can be compared.

8. Why is dew deposited more freely (*a*) on a lawn than on a gravel path; (*b*) on clear nights rather than on a windy cloudy night?

9. Why does an island enjoy a more equable climate than a place in the same latitude on a continent?

EXAMPLES XXV B (JOULE'S EQUIVALENT).

1. If $J = 1400$ ft.-lb., from what height must 20 lb. of water fall in order to raise its temperature by 1° C.?

2. If a steam engine raises, with uniform speed, a mass of 500 lb. through a height of 193 ft., how much heat is used in the process?

3. From what height must a piece of copper (S.H. $= 0{\cdot}1$) fall to the ground in order to raise its temperature by 1° C.? ($J = 1390$ ft.-lb.)

4. If an iron ball having a mass of 100 kilos falls from a height of 160 metres, how much ice at 0° C. could be melted by the energy developed?

5. If the energy liberated by the fall of a mass of 10 lb. from a height of 200 ft. could be used in raising the temperature of 20 lb. of ice at 20° F., how much would the temperature of the ice be raised? (S.H. of Ice $= {\cdot}5$. $J = 772$ ft.-lb.)

APPENDIX.

To find the coefficient of expansion of air at constant pressure.

In the method described in § 117, strong sulphuric acid is used to dry the air. This acid, being in contact with mercury, very slowly reacts with the metal and, as time elapses, a small quantity of sulphurous acid gas is liberated. If observations are taken soon after the higher temperature is reached, results show that the coefficient of expansion of this mixture of gases is the same as that of air. The advantage of the following method is that, as mercury is absent, no gas is liberated in drying the air.

117. Alternative method.

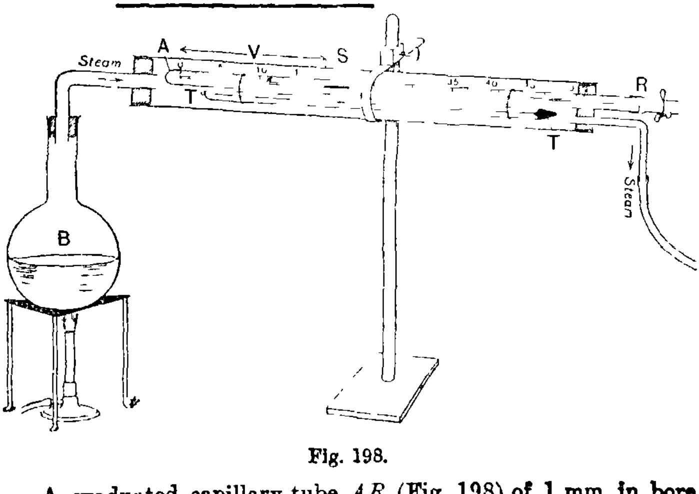


Fig. 198.

A graduated capillary tube AR (Fig. 198) of 1 mm. in bore and 52 cm. in length is heated gently and its open end is dipped into strong *sulphuric acid* coloured black with *indigo*. As the air in the tube cools and contracts a thread of sulphuric acid (S)

is drawn up the tube and encloses a small volume (V) of dry air. A thermometer (TT) is attached to the tube AR, and both are placed inside a wide glass tube of 2 cm. internal diameter, fitted with corks and clamped in a slightly sloping position. Steam is passed through the outer tube from a boiler B (see Fig. 198). The enclosed volume of air (V) is raised to the temperature of the steam and the thread of acid (S) is pushed along the capillary tube as the air expands. Readings are taken of the initial and final temperatures and volumes.

The following observations were obtained in actual experiments :—

Initial length of air column V_t	Final length of air column V_{100}	Initial temperature	Final temperature	Coefficient of expansion (for calculation see p. 194)
225·0 mm.	287·0 mm.	18·3° C.	100° C.	·00359
222·0 mm.	286·0 mm.	17·5° C.	100° C.	·00372
223·5 mm.	286·0 mm.	18·7° C.	100° C.	·00368
277·9 mm.	360·0 mm.	14·6° C.	99·8° C.	·00365
213·0 mm.	278·0 mm.	13·0° C.	100·3° C.	·00366

Note to Demonstrator. A piece of rubber tubing (R), fitted with a clip, is attached to the open end of the tube to *close it when not in use.* The tube AR may then be stored in any position for future use.

136. Bunsen's Ice Calorimeter (Fig. 164, p. 216) consists of a test-tube B fused into a closed glass vessel A, the latter being filled with water and connected to a mercury gauge CDE, of cross section a sq. cm. Before use the whole apparatus is cooled to 0° C. by placing it in a vessel containing melting ice. A *coating of ice is formed in A* around B by placing alcohol cooled to (say) $-20°$ C. in B or by blowing air through ether in B (§ 140). Having removed all trace of alcohol (or ether) from B, M grams of the *hot* body (at say 100° C.) are introduced thereby melting some of the ice in A. Since 1·09 c.c. of ice on melting becomes 1 c.c.

of water, there is a diminution of volume of $\cdot09$ c.c. per 1 gram of ice melted by the transference of $79\cdot5$ ($=L$) calories from the hot body. The volume of diminution (V) is found by noting the distance (d cm.) which the end of the mercury thread, E, recedes in the gauge, then $V = ad$ c.c.

$$\therefore \text{ grams of ice melted by the hot body} = \frac{ad}{\cdot09} = m.$$

$$\therefore \text{ heat lost by the hot body} = mL = \frac{ad}{\cdot09}L = MSt \text{ (see p. 216).}$$

In practice the actual volume of diminution is not calculated; the number of calories corresponding to a movement of unit distance of E on the gauge is obtained by a preliminary experiment of introducing water of known weight and at a known temperature into B.

REVISION PAPERS (HEAT).

OXFORD AND CAMBRIDGE LOCAL QUESTIONS.

PAPER A.

1. Describe the *construction* of a mercury thermometer, pointing out the chief difficulties that have to be overcome in the process.

2. State the requisites of a good thermometric substance. What properties of water render it unsuitable?

3. For what purposes would you use an alcohol thermometer in preference to a mercurial thermometer and *vice versa*?

How would the measure of specific heat be affected if the scale of temperature were changed from Centigrade to Fahrenheit?

4. Describe a thermometer which registers the lowest temperature attained during a given time.

5. What is the coefficient of linear expansion of a substance? Describe one method for determining its value practically. Give instances where manufacturers and engineers use or allow for the expansion and contraction of metals.

6. What are the relations between the coefficients of linear, areal and cubical expansion of a solid body? Prove your statement.

7. Describe any instance in which (*a*) heat is converted into work, (*b*) work into heat.

8. What is the effect of a rise in temperature on the time recorded by a clock which has a pendulum made entirely of steel?

Draw and describe some form of compensated pendulum, and explain the principle of its action.

Paper B.

1. How would you determine the "fixed points" of a thermometer, and graduate it? Would any correction be necessary if you did it on the top of a mountain or at the bottom of a mine?

2. How would you construct and graduate a simple form of air thermometer? Explain why air is not a convenient form of thermometric fluid for ordinary use.

3. Describe any method of finding the coefficient of expansion of a liquid.

4. In what respects does water differ from most other substances with regard to change of volume? Describe experiments which would show this difference.

5. State the law of the expansion of gases.

Describe an experiment for determining the coefficient of expansion of air, and state the numerical amount.

6. Why can a thin platinum wire be sealed into a glass tube so as to make an air-tight joint?

In what respects do gases when expanding under the application of heat behave differently from liquids and solids?

7. Draw and describe a thermometer for measuring maximum and minimum temperatures. Explain clearly how the instrument would behave during (*a*) a rise, (*b*) a fall, of temperature.

8. Heat is said to be a form of "Energy."

Explain the meaning of this statement, and give evidence in support of it.

Paper C.

1. State Boyle's Law and show how you would verify it for pressures greater or less than one atmosphere.

2. How is the Weight Thermometer used for finding the coefficient of expansion of a liquid?

3. Give an account of an experiment by which you could find the temperature at which water has its greatest density.

4. Give a definition of *temperature* and explain why it must be distinguished from *quantity of heat.*

5. State the principle of the "method of mixtures" in calorimetry. In actual practice what precautions are necessary in the determination of the specific heat of a metal by this method? Give reasons.

6. Define *water-equivalent* of a calorimeter. How would you arrange an experiment to find the W.E. of a calorimeter of unknown specific heat? What do you mean by *heat-capacity* of a calorimeter?

7. State the Laws of Fusion and explain how the latent heat of fusion of ice may be found experimentally.

8. State how you could show that a gas gains heat when compressed and loses heat when it expands.

Account for both facts.

PAPER D.

1. Define a *unit quantity of heat*. Describe any experiment which proves that different bodies which have the same mass and also the same temperature t and fall to the same temperature t' may give out very different quantities of heat in the process.

2. A quantity of water is cooled from 20° C. until it becomes ice below 0° C. Trace the changes in volume which occur as the temperature changes.

3. Explain how you would find the specific heat of a substance by the fusion of ice.

4. A heavy weight is attached to a loop of copper wire placed round the middle of a horizontal block of ice which is supported at each end. Describe and explain what happens.

Would any difference be noticed if a loop of strong silk thread were substituted for the copper wire? Give reasons.

5. State the Laws of Ebullition. Describe carefully an experiment to show the effect of the reduction of pressure on the boiling-point of water.

6. Describe a method of measuring the pressure of water-vapour at any temperature between 0° C. and 100° C.

7. What do you mean by Latent Heat of Vaporization of a substance? State how you would determine it experimentally.

8. What is the relation between Heat and Energy? Describe any experiment in which a measurable quantity of work is converted into a measurable quantity of heat.

Paper E.

1. Define the terms *Conduction*, *Convection*, and *Radiation*, and illustrate their meanings by reference to familiar phenomena.

2. Explain what is meant by maximum vapour pressure. How is the B. P. of water related to its V. P.? Describe an experiment showing this relation.

3. What are the chief differences between a saturated and unsaturated vapour? On what conditions does the deposition of dew depend?

4. Apply the theory of convection to explain (*a*) trade winds, (*b*) land and sea breezes, (*c*) the heating of buildings by hot water.

5. Define the terms: *calorie, specific heat, Latent Heat of Water, Latent Heat of Steam, mechanical equivalent of heat, dew-point.*

6. Describe an experiment for comparing the conducting powers of different materials. What uses are made of good and bad conductors?

7. What are the principal characteristics of the three states of matter?

State precisely what is meant by boiling-point and freezing-point. How are they affected by change of pressure? Explain the cause of the phenomenon known as *regelation*.

8. Two bulbs, *A* and *B*, connected by a tube, contain water and water vapour only, most of the water being in *B*. State and explain what happens when the bulb *A* is immersed in a freezing mixture of common salt and ice.

SENIOR LOCAL EXAMINATION QUESTIONS.

Thermometers.

1. Give the exact meaning of "degree Centigrade" and "thermal unit." Explain the difference between "temperature" and "quantity of heat." A thermometer is constructed so as to read 10° when placed in melting ice and 75° when immersed in steam from water boiling under the normal pressure. What Centigrade readings correspond to (*a*) $-5°$, (*b*) 25° on this thermometer?

2. Explain why the temperature of the *vapour* issuing from a boiling solution of salt is lower than the temperature of the *solution*. A thermometer registers 99·2° C. for the boiling-point of water when the barometer

stands at 74·5 cms. What correction, if any, must be applied to the thermometer reading? (A variation of 1 cm. from the normal height of the barometer alters the boiling-point of water by 0·37° C.)

3. A mercury thermometer and an alcohol thermometer agree in their readings at 0° C. and at 50° C. but differ at intermediate temperatures. To what causes may these differences be due? A thermometer registered 31·8° F. for the freezing-point and 212·4° F. for the boiling-point of water. The correct boiling-point under the observed pressure was 211·4° F. Find graphically or otherwise what corrections must be applied to a reading of 152° F., assuming that the bore is of uniform section.

4. A thermometer whose graduations indicate unknown but equal intervals of temperature is immersed in a bath with a Centigrade thermometer. The thermometer indicates 48° and 43° when the Centigrade thermometer indicates 58° and 48° respectively. What temperatures would the thermometer indicate at the freezing-point and boiling-point of water?

Coefficient of Expansion—Liquids.

5. Describe an experiment (preferably one you have performed) to determine the coefficient of *apparent expansion* of a liquid. State the chief precautions necessary to ensure accuracy.

A glass weight thermometer weighs 20 gm. when empty and 470 gm. when full of mercury at 0° C. After heating to 100° C., 6·85 gm. of mercury escape. Calculate the coefficient of cubical expansion of glass. (Coefficient of real expansion of mercury = ·000182.)

6. Explain the principle of a method of determining the coefficient of absolute expansion of turpentine (boiling-point of turpentine = 157° C.) by means of a U-tube. At what stage of the experiment should the height of the liquid in the cold limb be taken, and why?

If the height of one column at 0° C. is 61 cms. what will be the height of the other column at 100° C. if the coefficient of expansion of turpentine is ·00105?

7. Explain briefly how you would determine the coefficient of apparent expansion of turpentine (B.P. = 157° C.) between 0° C. and 100° C., and state in detail the practical precautions necessary for success. If the weight of liquid which fills a vessel at 0° C. is 50 gm. what weight of it will overflow at 100° C. if the coefficient of apparent expansion in this vessel is ·00102?

8. Explain how the coefficient of absolute expansion of a liquid can be obtained by using a weight thermometer. A quartz tube is exactly filled

with 100 gm. of mercury at 0° C., and when the temperature is raised to 100° C. it is found that a mass of 1·75 gm. of mercury overflows. If the coefficient of expansion of mercury is ·00018 per degree, find the coefficient of cubical expansion of the quartz.

9. Define coefficient of cubical expansion. What is meant by the statement that between 100° and 200° the mean coefficient of expansion of mercury is $\frac{1}{5374}$?

A weight thermometer contains at 0° C. 65·8 gm. of mercury. On the thermometer being heated to 100° C. one gram of mercury escapes. Assuming the coefficient of expansion of mercury between 0° and 100° is $\frac{1}{5550}$, calculate that of glass.

Specific Heat. Latent Heat.

10. What is meant by the "heat capacity" of a body? A piece of lead at 99° C. is placed in a calorimeter containing 200 gm. of water at 15° C. The temperature after stirring is 21° C. The calorimeter weighs 40 gm. and is made of material of specific heat = 0·1. Calculate the heat capacity of the piece of lead.

11. A substance at 98·6° C. weighing ·87 gm. was dropped into a Bunsen ice calorimeter. The diminution in volume observed was 7·9 c mm. Calculate the specific heat of the substance. What are the special advantages of Bunsen's calorimeter? (Specific gravity of ice = ·917; latent heat of ice = 80.)

12. An iron vessel weighing 7·5 gm. contains 15 gm. of a body which melts at 59° C. and which has a specific heat of ·75 when liquid and ·32 when solid. If the latent heat of fusion of the body is 94, calculate the quantity of heat lost by the vessel and its contents in cooling from 90° C. to 40° C. (Specific heat of iron = ·11.)

13. Describe how to measure the specific heat of (*a*) ice, (*b*) steam. Give a detailed account with full explanations of the mode of procedure followed in determining the specific heat of a liquid by the method of cooling

14. Explain how the specific heat of a substance is found by Bunsen's calorimeter.

Describe an ice-block calorimeter, and state the relative advantages of Bunsen's.

15. Explain the nature of the latent heat of steam. Assuming that the total quantity of heat, in calories, required to raise 1 gm. of water in temperature from 0° to T° and to convert it into steam at T° is 606·5 + ·305T, calculate the latent heat of steam at 40°.

CONDUCTIVITY.

16. Explain the meaning of the statement: the mechanical equivalent of heat is 1400 foot-lb. From what height must 20 lb. of water fall in order to raise its temperature by 1° C., assuming that no heat is lost?

17. Describe a practical method of determining the thermal conductivity of liquid mercury.

A plate of copper 0·5 cm. thick, and 100 sq. cm. in area, is in contact with steam at 100° C. on one side and with melting ice on the other. Calculate (*a*) how much steam will be condensed in an hour, (*b*) how much ice will be melted in the same time?

Conductivity of copper in C.G.S. units = 1·108 (calories). Latent heat of water = 80. Latent heat of steam = 540.

18. The temperature of one face of an iron plate is kept at 100° C., while that of the opposite face is 60° C.; the thickness of the plate is ·5 cm. and the specific conductivity of the substance is ·12 calorie. Find in calories the total quantity of heat which in 30 mins. passes through each sq. cm. of the plate.

19. What is the measure of the conducting power of a body for heat? The thermal conductivity of iron is ·16. An iron boiler has a surface of 4 sq. metres and a thickness of ·75 cm. The water in the boiler is kept at 100° C. and the heating surface at 260° C. Find the number of kilograms of water evaporated in 30 mins. if latent heat of steam is 536 cal. per gm.

MECHANICAL EQUIVALENT OF HEAT.

20. Describe two distinct experiments in support of the statement, "heat is a form of energy."

From what height must a piece of copper (S.H. = ·1) fall to the ground in order to raise its temperature by 1° C., assuming that it imparts no heat to anything else? (The mechanical equivalent of heat is 1390 when the pound, foot and Centigrade degree are the units of measurement.)

21. What is meant by the "mechanical equivalent of heat"? Assuming that the mechanical equivalent of heat is 42×10^6 ergs and that the specific heat of copper is ·1, and the melting-point of copper 1080° C., calculate with what velocity a bullet of copper must strike a non-conducting target in order that the heat developed may be just sufficient to raise the temperature of the metal from 0° C. to its melting-point.

22. Explain what is meant by the statement that the mechanical equivalent of heat is 42×10^6 ergs per calorie.

If a petrol engine converts into work 30 % of the heat supplied, find what power it will develop if it uses 100 c.c. of petrol per minute and each c.c. of petrol when burnt liberates 8000 calories. (A horse-power $= 8 \times 10^9$ ergs per second.)

23. Explain what is meant by the specific heats of a gas at constant pressure and at constant volume. State which of these constants is the greater and give reasons for your answer.

The capacity for heat of 22·4 litres of hydrogen at 760 mm. pressure and 0° C. is 4·8 calories per degree Centigrade, when volume is kept constant. Calculate the heat absorbed when the same volume of hydrogen is raised in temperature 1° C. at constant pressure, it being assumed that the energy of a gas at the same temperature is constant. (Coefficient of expansion of hydrogen $= \frac{1}{273}$. Density of mercury = 13·6. Gravity = 981 c.g.s. units. Mech. equivalent of heat $= 42 \times 10^6$ ergs.)

GENERAL.

24. How would you compare the absorbing powers of two partially transparent substances, such as glass and quartz, for radiant heat?

It is found that a beam of radiant heat of a definite wave length loses $\frac{1}{10}$ of its energy in passing through 1 mm. of glass. What will be the intensity of the beam after it has passed through 3 mm. of glass?

25. Explain the following facts:

(*a*) When water is suddenly heated in a flask the level of the water falls.

(*b*) A piece of copper becomes hotter than the same weight of water if they are heated equally for the same length of time.

(*c*) Kneading snow makes it "bind" into a firm snowball.

(*d*) On a cold evening moisture is deposited on the windows of a warm room.

ANSWERS TO EXAMPLES.

Examples I a.

6. (a) 3700·1 m., (b) 3·7001 km. 7. 500. 8. 2·54 cm.; 0·3937 in.
9. 29·92 in. 10. 2600 mm. 11. 3578 75 m. 12. $3\frac{1}{2}$ hrs.
13. 20 m. 14. 2·54 cm. 15. 0·12 mm.

Examples I b.

1. $8\frac{1}{7}$. 2. (a) 22 in., (b) 11 cm., (c) 15·84 dm. 3. (a) 14 yds., (b) 4·9 cm.
4. (a) 29·75 yds., (b) 6·25 mm. 5. 58. 6. 28 in. 7. 44.
8. 6228 mi. 9. 22 yds. 10. $7\frac{1}{2}$.

Examples II a.

3. 5,000,105 sq. cm. 4. 6·4516 sq. cm. 5. 1·196 sq. yds.
6. 972 sq. cm.; 0·0972 sq. m. 7. 50. 8. 32 yds. 9. 42 sq. cm.
10. 30 87 sq. in.; 199·92 sq. cm.; 6·47 sq. cm.
11. (a) 14·35 sq. cm., (b) 63·5 sq. cm.; (c) $\frac{x^2}{4}$ sq. cm.
12. (a) 3·5 cm., (b) 5·5 cm., (c) 12 in. 13. Height = 15 cm.; Base = 5 cm.
14. 176 sq. cm. 15. 2·325 lb.

Examples II b.

1. (a) $50\frac{2}{7}$ sq. cm., (b) 6·16 sq. in., (c) 0·9856 sq. dm.
2. (a) 7 cm., (b) 10 in., (c) 2·1 dm.
3. (a) 154 sq. in., (b) 55 44 sq. cm., (c) 38·5 sq. cm.
4. (a) 87·12 sq. cm., (b) 440 sq. in., (c) 422·4 sq. in.
5. (a) 8 cm., (b) 2240 cm., (c) 8 cm.
6. (a) 110 sq. in., (b) $47\frac{1}{7}$ sq. cm., (c) 22·5 sq. cm.
7. (a) 22·91 sq. in., (b) 254·59 sq. in. 8. 1578 5 sq. yds. 9. 770 lb.
10. $653\frac{5}{7}$ sq. ft.

EXAMPLES III A.

6. 16387·064 c.mm. **7.** 28·316 c.dm.
8. (*a*) 24·96 cu. in., (*b*) 192·24 c.c. **9.** (*a*) 4·2 in., (*b*) 1·5 m.
10. (*a*) 18·15 c.c., (*b*) 29·337 c.c. **12.** 11750 c.c. **13.** 54 c.c.
14. 160 l. **15.** 50 cm. **16.** 44 days.

EXAMPLES III B.

1. $111\frac{1}{9}$ sq. mm. **2.** 12·5 m. **3.** 17·5 cm. **4.** 0·1 cm.
5. Height = 10 cm.; Diameter = 14 cm. **6.** (*a*) $157\frac{1}{7}$ c.c., (*b*) 46·2 c.c.
7. 33 c.c. **8.** 12 cm. **9.** 38,808 cu. ft. **10.** $1437\frac{1}{3}$ c.c.
11. 1·44 mm. **12.** 3 : 1 : 2. **13.** 12,728,571 cu. ft. **14.** $170\frac{3}{4}$ cu. in.
15. 384 c.c.

EXAMPLES IV.

6. 1664·535 gm.; 1·664535 kgm. **7.** 4 gm.; 0·25 cm. **8.** 3·5 mm.
9. (*a*) 453·6 gm., (*b*) 2·204 lb. **10.** 154 lb. **11.** 139 c.c.
12. 20 gm. **13.** 28 gm. **14.** 3·1185 gm. **15.** 29 l. **16.** 4·536 l.

MISCELLANEOUS EXAMPLES, CHS. I—IV.

1. 500 cm. **2.** 8·8 sq. cm. **3.** $300\frac{2}{7}$ sq. cm. **4.** 100; $471\frac{3}{7}$ sq. yds.
5. 3 ft. **6.** 1·609 km. **7.** 3850 sq. yds. **8.** 19 mgm.
9. 1 mm. **10.** $12\frac{1}{2}$ gal. **11.** 100. **12.** 4·8 sq. cm.
13. 105 sq. in. **14.** 5; 37. **15.** 3375 lb. **16.** 96 km.
17. 18·9 sq. cm. **18.** 45 sq. in. **19.** 0·12 mm. **20.** $2\frac{6}{11}$ ft.
21. 981·456 cm. **22.** 15 sq. ft. 40 sq. in. **23.** 165,600 cu. m.
24. $102\frac{6}{7}$ c.c. **25.** 264 cu. yds. **27.** 300 cu. ft. **28.** £2. 4*s*. 0*d*.
29. 13·9 cu. ft. **30.** 5·57 sc.

EXAMPLES V A.

4. 0·5 gm. per c.c. **5.** 19·3 gm. per c.c. **6.** 11·98 c.c.
7. 39·6 gm. **8.** 0·25 sq. cm. **9.** 0·04 cm. **10.** 1·114 gm. per c.c.
11. 5 cm. **12.** 1 cm.

EXAMPLES V B.

4. 0·88 gm. per c.c. **5.** 0·826. **6.** 60 gm. **7.** 86 gm.
8. 1·01 gm. per c.c. **9.** 320 c.c. **10.** 16 gm. **11.** 0·926 gm. per c.c.
12. 1·12 gm. per c.c. **13.** 85 gm.

Examples V c.

1. 7·2. 2. 2·5. 3. 86 gm. 4. 1·6. 5. 11·4.

Examples VII.

1. 11 cm. 2. 1·46 gm. 3. 39 325 gm. 4. 1·6 cm.
5. 1033·6 gm. 6. 96 kgm.; 40 gm. 7. 250 lb. 8. 2·5 cm.
9. 69·7 kgm.; 51·25 m. 10. 34·56 ft. 11. 0·91. 12. 10 cm.
13. 50 cm.

Examples VIII.

1. 8·9. 2. 59·94 gm. 3. 2·7 gm. per c.c. 4. 124 kgm.
5. 2·5 gm. per c.c. 6. 10·5. 7. 40·61 gm. 8. 160 gm.; 96 gm.
9. 3 c.c. 10. 189 lb. 11. 7·3 gm. per c.c. 12. 2·1 gm. per c.c.
13. 10·24 gm. 14. 1·4 cm

Examples IX.

1. 5·7 c.c.; 4·3 gm. 2. 0·51. 3. [illegible]. 4. 4789·7 cu. yds.
5. 15 cm.; 0·878. 6. 8 c.c. 7. 716,800 cu. ft. 8. 1·512 ft.
9. 1·12; 16·47 cm. 10. 1·2. 11. 8·9. 12. 10·1 gm.

Examples X.

1. 0·00125 gm. per c.c.; 71·24 gm. 2. 201·6 kgm. 3. 1·27 gm.
4. 81·293 gm. 5. 12·03 gm. 6. 0·819. 7. 28·7 cm.
8. 47·5 c.c. 9. 15·2 cm.; $70\frac{2}{3}$ cm. 10. 380 c.c.; 346·75 c.c.

Examples XI.

1. 10·5; 2·5; 390 gm. 2. 18·95 gm. 3. 40 gm. 4. 2·5; [illegible]
5. 0·48. 7. 2; 0·9. 8. 0·25. 10. 26·[illegible] in.
11. 10·35 gm. per c.c. 12. 5 c.c.; 12·5 gm. 13. 0·819.
14. 952 gm. 15. 90 lb. 16. 19·3. 17. 1020 cm.; 816 cm.
18. 0·55 lb.; 57 lb. 19. 1·12. 20. 11 ft. 21. 59·4 gm.; 9·9 gm.
22. 1026 lb. 23. 25·36 gm. 24. 2·985. 25. 80 in.

Examples XII.

1. 44 ft. per sec. 2. 60 miles per hr.
3. 52 miles per hr.; $76\frac{4}{15}$ ft. per sec. 4. 393 miles. 5. 36 miles per hr.
6. $11\frac{2}{3}$ miles. 7. 4 ft.-sec. units; 24 ft. per sec. 8. 3 sec.
9. 20 ft. per sec.; 35 ft. per sec. 10. 8 ft.-sec. units; 200 ft.
11. 80 ft. per sec.; $2\frac{1}{2}$ sec. 12. $1\frac{1}{4}$ sec. 13. 1600 ft.; 1 sec.
14. 112 ft. 15. 3 sec. after 1st is thrown; 48 ft. from ground.

Examples XIII.

4. 2 lb. wt. 5. 5·6 lb. wt. at $\angle 45°$ with 8 lb. wt.
6. 10 lb. wt. at $\angle 37°$ with 8 lb. wt. 7. 15 lb. wt.
8. 25 lb. wt. at $\angle 37°$ with vertical. 9. 9·9 lb. wt. at $\angle 45°$ with string.
10. 5 lb. wt.; 8·7 lb. wt. 11. 14 lb. wt.

Examples XIV A.

1. 7 lb. 2. 4 ft. from 9 lb. 3. 1 ft. 4. 1 ft. 8 in. 5. 80 gm.
6. 1 ft. from 4 lb. wt. 7. 90 gm. 8. 15 lb.

Examples XIV B.

1. (*a*) 12 lb. wt.; 5 in. from 7 lb. wt., (*b*) 70 gm. wt.; 4 cm. from 50 gm. wt.
2. (*a*) 2 lb. wt.; 27 in. from 11 lb. wt., (*b*) 10 gm. wt.; 10 cm. from 20 gm. wt.
3. 8 lb. wt.; 3 ft. from *Q*. 4. (*a*) 8 lb. wt., (*b*) 6 lb. wt.
5. (*b*) $83\frac{1}{3}$ lb.; $16\frac{2}{3}$ lb. 6. 5 lb.; 6 lb. 7. 1 ft. 4 in. from man.
8. 3 ft.; 1 ft. 9. $2\frac{1}{4}$ ft.

Examples XIV C.

1. 1 ft. 10 in. from 2 lb. wt. 2. 47·5 cm. 3. 2 in. from centre.
4. 1·9 in. from centre of *AB*. 5. $4\frac{1}{3}$ in. 7. $\frac{2}{3}$ in. from centre of square.
8. $\frac{6}{11}$ in. 9. $\frac{1}{12}$ diameter from centre. 10. $\frac{3}{8}$ in. from centre of disc.

Examples XV A.

1. 1 yd. 2. 55 lb. 3. 50 lb. wt. 4. 504 lb. wt.
5. $2\frac{1}{2}$ lb. wt. 6. 16 lb. wt. 7. 120 lb. 8. 1 ft. 8 in.
9. 8000 lb. 10. (*a*) 5 lb. wt., (*b*) 4·2 in., (*c*) 5.

Examples XV B.

1. 14 lb. wt.; 8. 2. 4. 3. $1\frac{7}{8}$ tons. 4. 14 lb. wt.
5. (*a*) $12\frac{1}{2}$ lb. wt., (*b*) 10 lb. wt. 6. (*a*) 160 lb., (*b*) 156 lb.
7. (*a*) Number = 4; Weight = 2 lb. 8. $102\frac{3}{4}$ lb. 9. 120 lb.
10. 16 lb. wt. 11. 1500 lb. 12. (*a*) 8 in., (*b*) 7 in.

Examples XV C.

1. 6 lb. wt. 2. (*a*) 200, (*b*) $200\sqrt{2}$, (*c*) $200\sqrt{3}$ gm. wt. 3. $41\frac{2}{3}$ lb.
4. (*a*) 10 lb. wt., (*b*) $\frac{20}{\sqrt{3}}$ lb. wt.
5. (*a*) $20\sqrt{3}$ lb. wt., (*b*) 60 lb. wt., (*c*) $60\sqrt{3}$ lb. wt. 6. $8\frac{1}{3}$ lb.

EXAMPLES XV D.

1. 4 lb. wt. 2. 13 lb. wt. 3. 150 gm. wt.

EXAMPLES XVI A.

1. (*a*) 2 ft.-sec. units, (*b*) 64 ft.-sec. units. 2. 25 lb. wt. 3. $6\frac{1}{4}$ lb.
4. 80 pdls. 5. 72 ft. 6. 13,826 dynes. 7. 4 cm.-sec. units.
8. 20 cm. per sec. 9. 98·1 cm.-sec. units. 10. 39·24 m.
11. (*a*) 20 pdls., (*b*) $\frac{5}{8}$ lb. wt. 12. (*a*) 25,667 lb. wt., (*b*) 14,000 lb. wt.

EXAMPLES XVI B.

1. (*a*) 150, (*b*) 11, (*c*) 19,712,000. 2. 10 ft. per sec. 3. 44.
4. (*a*) 525 pdls., (*b*) 5·25. 5. (*a*) 11·8125, (*b*) $118\frac{1}{8}$ pdls.
6. $6\frac{3}{4}$ mi. per hr. 7. $13\frac{1}{3}$ ft. per sec.; $8\frac{2}{3}$ ft. per sec.

EXAMPLES XVI C.

1. 2000 ft.-pdls. 2. 500 ergs. 3. (*a*) 16,000, (*b*) 500.
4. 102·7 lb. per ton. 5. (*a*) $8\frac{13}{32}$ lb. wt., (*b*) $98\frac{7}{18}$ lb. wt.

PAPER A.

1. 18 ft. per sec.; 20° to AB. 2. (*a*) 3 ft.-sec. units, (*b*) 200 ft.
3. 6 ft.-sec. units. 4. 4 sec. 5. 116 in. 7. 20 ft.-lb.
8. 1 sec. after shot is fired; 1024 ft. from each. 9. (*a*) 9 : 10, (*b*) 27 : 10.

PAPER B.

1. 80 lb. wt. 2. 90 lb. wt. 3. 14 gm. wt.; 6 cm. from 26 gm. wt.
4. $2\frac{1}{3}$ cm. 6. 4 lb. wt. upwards on C; 16 lb. wt. downwards on D.
8. $\frac{10}{\sqrt{3}}$ lb. wt. 9. 1·6 oz. wt. in AB; 1·2 oz. wt. in AC.
10. $5\sqrt{3}$ perpendicular to AB.

EXAMPLES XVII (THERMOMETRY).

1. 16° C.; 60·8° F. 2. 27°. 3. 59° F.; −4° F.; 101° F.; 35·6° F.
4. 40° C. 5. −40°. 6. 2·5°. 7. 20° C.; −5° C.; 90° C.; −25° C.
8. 40°. 9. 160° C. 10. 671·6° F.; −38·2° F. 11. −80°.
12. 10° C. 13. −130° C.; 78·3° C.

Examples XVIII (Expansion of Solids).

1. ·000012. 2. ·006 yd. 3. 200° C. 4. 100 cm.
5. 200 057 cm. 6. ·000012. 7. $1\frac{1}{28}$ ft. 8. 20·019 cm.
9. 286·7° C. 10. ·0000086. 11. 1·6 ft. 12. ·0000117.
13. 50·17 sq. ft. 14. 60 522 cu. ft. 15. ·000019. 16. 22044·88 c.c.
17. 2·00516 litres. 18. 11·3 gm. per c.c.

Examples XIX a (Expansion of Liquids).

1. 201·24 c.c. 2. 35° C. 3. ·001. 4. 60 gm. 5. 100 c.c.
6. 3 05 gm. 7. ·001. 8. 10·38 gm. 9. ·00038. 10. 1000 c.c.
11. ·00125. 12. 7·16 c.c. 13. ·000026. 14. 13·36 gm. per c.c.
15. ·001. 16. ·0009.

Examples XIX b (Expansion of Gases).

1. 182 c.c. 2. $2\frac{2}{7}$ gm. 3. $\frac{1}{273}$. 4. 119 c.c. 5. 91° C.
6. 546° C. 7. ·00364. 8. 91 c.c. 9. 0 c.c. 10. ·00372.
11. 455 c.c. 12. 600 c.c. 13. 152 cm. 14. 91·2 cm.
15. 54·6 c.c. 16. 1071·05 c.c. 17. 273 c.c.

Examples XX a (Calorimetry).

1. 50,000 cals. 2. 125 gm. 3. 1600 cals. 4. (*a*) 20 cals., (*b*) 22 cals.
5. 32·1° C. 6. 55° C. 7. 80 gm. 8. 200 cals.; 5 cals.; 5 gm.
9. 87° C. 10. 20 cals.; 2 gm. 11. 27·5° C. 12. 60 gm.

Examples XX b (Specific Heat).

1. (*a*) 760 cals., (*b*) 20°, (*c*) 80 gm. 2. ·095. 3. 66 gm. 4. 20° C.
5. 853° C. 6. 4 gm. 7. 43 6° C. 8. ·2. 9. ·055. 10. 3449.
11. ·6. 12. ·096.

Examples XXI (Latent Heat of Fusion).

1. 5 gm. 2. 1900 cals. 3. 1100 cals. 4. 800 gm. 5. 20° C.
6. 30 gm. 7. 40 gm. 8. 31·4° C. 9. 80 cals. 10. 80 cals.
11. 10 gm. 12. 80 cals. 13. 9·5 gm. 14. 2657 cals.
15. 75·2° C. 16. 0·5 c.c. 17. 80 cals. 18. 1000° C.
19. 3326·5 cals. 20. 18·8° C. 21. 800 cals.

Examples XXII (Latent Heat of Vaporization).

1. 536 cals. 2. 2680 cals. 3. 12·5 gm. 4. 31,300 cals.
5. 53·6 gm. 6. 11 gm. 7. 1 gm. 8. 41·3° C. 9. 12·5° C.
10. 536·7 cals. 11. 536 cals. 12. 536 cals. 13. 7160 cals.
14. 7210 cals. 15. 63·2° C. 16. 540 cals. 17. 2389 cals.
18. 34° C. 19. 29·54 gm.

Examples XXIII a (Vapour Pressure).

3. 449·6 c.c. 7. ·25° C.

Examples XXV b (Joule's Equivalent).

1. 1400 ft. 2. 69 lb. deg. C. units if $J = 1400$. 3. 139 ft.
4. 467 gm. 5. ·26° F.

SENIOR LOCAL EXAMINATION QUESTIONS.

1. (*a*) $-23\frac{1}{13}$, (*b*) $23\frac{1}{13}$. 2. Add ·245°. 3. Subtract ·6° F.
4. F.P. $= 19°$, B.P. $= 69°$. 5. ·000027. 6. 67·405 cm.
7. 4·63 gm. 8. ·000002. 9. ·0000267. 10. 15·69 cals.
11. ·0814. 12. 1891·2 cals. 15. 578·7 cals. 16. 1400 ft.
17. (*a*) 147·73 kilos, (*b*) 997·2 kilos. 18. 17280 cals. 19. 4585 kgm.
20. 139 ft. 21. $36000\sqrt{7}$ cm. per sec. 22. 21 H.P.
23. 6·78 cals. 24. ·729.

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CAMBRIDGE : PRINTED BY W. LEWIS AT THE UNIVERSITY PRESS